16

Compressible Flow

16.1. INTRODUCTION

A compressible flow is that flow in which the *density of the fluid changes during flow.* All real fluids are compressible to some extent and therefore their density will change with change in pressure or temperature. If the relative change in density $\Delta\rho/\rho$ is small, the fluid can be treated as incompressible. A compressible fluid, such as air, can be considered as incompressible with constant ρ if *changes in elevation are small, acceleration is small,* and/or *temperature changes are negligible.* In other words, if Mach's number U/C, where C is the sonic velocity, is small, compressible fluid can be treated as incompressible.

• The gases are treated as compressible fluids and study of this type of flow is often referred to as *'Gas dynamics'.*

• Some important problems where *compressibility effect* has to be considered are :

(*i*) Flow of gases through nozzles, orifices ;

(*ii*) Compressors ;

(*iii*) Flight of aeroplanes and projectiles moving at higher altitudes ;

(*iv*) Water hammer and accoustics.

• *'Compressibility' affects the drag coefficients of bodies by formation of shock waves, discharge coefficients of measuring devices such as orificemeters, venturimeters and pitot tubes, stagnation pressure and flows in converging-diverging sections.*

16.2. BASIC EQUATIONS OF COMPRESSIBLE FLUID FLOW

The basic equations of compressible fluid flow are : (*i*) *Continuity equation,* (*ii*) *Momentum equation,* (*iii*) *Energy equation,* and (*iv*) *Equation of state.*

16.2.1. Continuity Equation

In case of *one-dimensional flow,* mass per second = ρAV

(where ρ = mass density, A = area of cross-section, V = velocity)

Since the mass or mass per second is constant according to law of conservation of mass, therefore,

$$\rho AV = \text{constant} \qquad ...(16.1)$$

Differentiating the above equation, we get

$$d(\rho AV) = 0 \text{ or } \rho d(AV) + AVd\rho = 0$$

or

$$\rho(AdV + VdA) + AVd\rho = 0 \text{ or } \rho AdV + \rho VdA + AVd\rho = 0$$

Dividing both sides by ρAV and rearranging, we get

$$\frac{d\rho}{\rho} + \frac{dA}{A} + \frac{dV}{V} = 0 \quad ...(16.2)$$

Eqn. (16.2) is also known as equation of continuity in *differential form.*

16.2.2. Momentum Equation

The momentum equation for compressible fluids is similar to the one for incompressible fluids. This is because in momentum equation the *change in momentum flux is equated to force required to cause this change.*

Momentum flux = mass flux × velocity = $\rho AV \times V$

But the mass flux *i.e.*, ρAV = constant ...By continuity equation

Thus the momentum equation is completely independent of the compressibility effects and for compressible fluids the momentum equation, say in X-direction, may be expressed as :

$$\Sigma F_x = (\rho AVV_x)_2 - (\rho AVV_x)_1 \quad ...(16.3)$$

16.2.3. Bernoulli's or Energy Equation

As the flow of compressible fluid is steady, the Euler equation is given as :

$$\frac{dp}{\rho} + VdV + gdz = 0 \quad ...(16.4)$$

Integrating both sides, we get

$$\int \frac{dp}{\rho} + \int VdV + \int gdz = \text{constant}$$

or

$$\int \frac{dp}{\rho} + \frac{V^2}{2} + gz = \text{constant} \quad ...(16.5)$$

In compressible flow since ρ is not constant it cannot be taken outside the integration sign. In compressible fluids the pressure (p) changes with change of density (ρ), depending on the type of process. Let us find out the Bernoulli's equation for isothermal and adiabatic processes.

(*a*) **Bernoulli's or energy equation for isothermal process :**

In case of an isothermal process,

$$pv = \text{constant or } \frac{p}{\rho} = \text{constant} = c_1 \text{ (say)}$$

(where v = specific volume = $1/\rho$)

$$\therefore \quad \rho = \frac{p}{c}$$

Hence

$$\int \frac{dp}{\rho} = \int \frac{dp}{p/c_1} = \int \frac{c_1 dp}{p} = c_1 \int \frac{dp}{p} = c_1 \log_e p = \frac{p}{\rho} \log_e p \quad \left(\because \; c_1 = \frac{p}{\rho}\right)$$

Substituting the value of $\int \frac{dp}{p}$ in eqn. (16.5), we get

$$\frac{p}{\rho} \log_e p + \frac{V^2}{2} + gz = \text{constant}$$

Dividing both sides by g, we get

$$\frac{p}{\rho g}\log_e p + \frac{V^2}{2g} + z = \text{constant} \qquad ...(16.6)$$

Eqn. (16.6) is the Bernoulli's equation for compressible flow undergoing isothermal process.

(*b*) **Bernoulli's equation for adiabatic process :**

In case of an adiabatic process,

$$pv^\gamma = \text{constant or } \frac{p}{\rho^\gamma} = \text{constant} = c_2 \text{ (say)}$$

$$\therefore \qquad \rho^\gamma = \frac{p}{c_2} \text{ or } \rho = \left(\frac{p}{c_2}\right)^{1/\gamma}$$

Hence $\int \frac{dp}{\rho} = \int \frac{dp}{(p/c_2)^{1/\gamma}} = = (c_2)^{1/\gamma} \int \frac{1}{p^{1/\gamma}}\, dp = (c_2)^{1/\gamma} \int p^{-1/\gamma}\, dp$

$$= (c_2)^{1/\gamma}\left[\frac{p^{-\frac{1}{\gamma}+1}}{\left(-\frac{1}{\gamma}+1\right)}\right] = \frac{(c_2)^{1/\gamma}(p)^{\left(\frac{\gamma-1}{\gamma}\right)}}{\left(\frac{\gamma-1}{\gamma}\right)} = \frac{\gamma}{\gamma-1}(c_2)^{1/\gamma}(p)^{\left(\frac{\gamma-1}{\gamma}\right)}$$

$$= \left(\frac{\gamma}{\gamma-1}\right)\left(\frac{p}{\rho^\gamma}\right)^{1/\gamma}(p)^{\left(\frac{\gamma-1}{\gamma}\right)} \qquad \left(\because\ c_2 = \frac{p}{\rho^\gamma}\right)$$

$$= \left(\frac{\gamma}{\gamma-1}\right)\left(\frac{p^{1/\gamma}}{\rho^{\gamma\times\frac{1}{\gamma}}}\right)(p)^{\left(\frac{\gamma-1}{\gamma}\right)}$$

$$= \left(\frac{\gamma}{\gamma-1}\right)\frac{(p)^{\left(\frac{1}{\gamma}+\frac{\gamma-1}{\gamma}\right)}}{\rho} = \left(\frac{\gamma}{\gamma-1}\right)\frac{p}{\rho}$$

Substituting the value of $\int \frac{dp}{p}$ in eqn. (16.6), we get

$$\left(\frac{\gamma}{\gamma-1}\right)\frac{p}{\rho} + \frac{V^2}{2} + gz = \text{constant}$$

Dividing both sides by g, we get

$$\left(\frac{\gamma}{\gamma-1}\right)\frac{p}{\rho g} + \frac{V^2}{2g} + z = \text{constant} \qquad ...(16.7)$$

Eqn. (16.7) is the Bernoulli's equation for compressible flow undergoing adiabatic process.

Example 16.1. *A gas with a velocity of 300 m/s is flowing through a horizontal pipe at a section where pressure is 78 kN/m² absolute and temperature 40°C. The pipe changes in diameter and at this section, the pressure is 117 kN/m² absolute. Find the velocity of the gas at this section if the flow of the gas is adiabatic. Take R = 287 J/kg K and γ = 1.4.*

(Punjab University)

Sol. Section 1 : Velocity of the gas, $V = 300$ m/s

Pressure, $p_1 = 78$ kN/m^2

Temperature, $T_1 = 40 + 273 = 313$ K

Section 2 : Pressure, $p_2 = 117$ kN/m^2

$R = 287$ J/kg K, $\gamma = 1.4$

Velocity of gas at section 2, V_2 :

Applying Bernoulli's equations at sections 1 and 2 for *adiabatic process*, we have

$$\left(\frac{\gamma}{\gamma-1}\right)\frac{p_1}{\rho_1 g}+\frac{V_1^2}{2g}=z_1=\left(\frac{\gamma}{\gamma-1}\right)\frac{p_2}{\rho_2 g}+\frac{V_2^2}{2g}+z_2 \quad \text{[Eqn. (16.7)]}$$

But $z_1 = z_2$, since the pipe is horizontal.

$$\therefore \quad \left(\frac{\gamma}{\gamma-1}\right)\frac{p_1}{\rho_1 g}+\frac{V_1^2}{2g}=\left(\frac{\gamma}{\gamma-1}\right)\frac{p_2}{\rho_2 g}+\frac{V_2^2}{2g}$$

Cancelling 'g' on both sides, we get

$$\left(\frac{\gamma}{\gamma-1}\right)\left(\frac{p_1}{\rho_1}-\frac{p_2}{\rho_2}\right)=\frac{V_2^2}{2}-\frac{V_1^2}{2}$$

or,

$$\left(\frac{\gamma}{\gamma-1}\right)\frac{p_1}{\rho_1}\left(1-\frac{p_2}{\rho_2}\times\frac{\rho_1}{p_1}\right)=\frac{V_2^2}{2}-\frac{V_1^2}{2}$$

$$\therefore \quad \left(\frac{\gamma}{\gamma-1}\right)\frac{p_1}{\rho_1}\left(1-\frac{p_2}{p_1}\times\frac{\rho_1}{\rho_2}\right)=\frac{V_2^2}{2}-\frac{V_1^2}{2} \quad ...(i)$$

For an adiabatic flow : $\dfrac{p_1}{\rho_1^{\gamma}}=\dfrac{p_2}{\rho_2^{\gamma}}$ or $\dfrac{p_1}{p_2}=\left(\dfrac{\rho_1}{\rho_2}\right)^{\gamma}$ or $\dfrac{\rho_1}{\rho_2}=\left(\dfrac{p_1}{p_2}\right)^{\frac{1}{\gamma}}$

Substituting the value of $\dfrac{\rho_1}{\rho_2}$ in eqn (i), we get

$$\left(\frac{\gamma}{\gamma-1}\right)\frac{p_1}{\rho_1}\left\{1-\frac{p_2}{p_1}\times\left(\frac{p_1}{p_2}\right)^{\frac{1}{\gamma}}\right\}=\frac{V_2^2}{2}-\frac{V_1^2}{2}$$

$$\left(\frac{\gamma}{\gamma-1}\right)\frac{p_1}{\rho_1}\left\{1-\left(\frac{p_2}{p_1}\right)^{1-\frac{1}{\gamma}}\right\}=\frac{V_2^2}{2}-\frac{V_1^2}{2}$$

or,

$$\left(\frac{\gamma}{\gamma-1}\right)\frac{p_1}{\rho_1}\left\{1-\left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}}\right\}=\frac{V_2^2-V_1^2}{2} \quad ...(ii)$$

At section 1 :

$$\frac{p_1}{\rho_1}=RT_1=287\times313=89831$$

$$\frac{p_2}{p_1}=\frac{117}{78}=1.5, \text{ and } V_1=300 \text{ m/s}$$

Substituting the values in eqn. (*ii*), we get

$$\left(\frac{1.4}{1.4-1}\right) \times 89831 \left\{1-(1.5)^{\frac{1.4-1}{1.4}}\right\} = \frac{V_2^2}{2} - \frac{300^2}{2}$$

$$314408.5\,(1 - 1.1228) = \frac{V_2^2}{2} - 45000 \text{ or } - 38609.4 = \frac{V_2^2}{2} - 45000$$

or, $$V_2^2 = 12781.2 \text{ or } \mathbf{V_2 = 113.05\ m/s.}\quad \textbf{(Ans.)}$$

Example 16.2. *In the case of air flow in a conduit transition, the pressure, velocity and temperature at the upstream section are 35 kN/m², 30 m/s and 150°C respectively. If at the downstream section the velocity is 150 m/s, determine the pressure and the temperature if the process followed is isentropic. Take* $\gamma = 1.4$, $R = 290$ *J/kg K.*

Sol. *Section 1 (upstream) :* Pressure, $p_1 = 35$ kN/m²,

Velocity, $V_1 = 30$ m/s,

Temperature, $T = 150 + 273 = 423$ K

Velocity, $V_2 = 150$ m/s

$R = 290$ J/kg K, $\gamma = 1.4$

Section 2 (downstream) :

Pressure, p_2 :

Applying Bernoulli's equation at sections 1 and 2 for isentropic (reversible adiabatic) process, we have

$$\left(\frac{\gamma}{\gamma-1}\right)\frac{p_1}{\rho_1 g} + \frac{V_1^2}{2g} + z_1 = \left(\frac{\gamma}{\gamma-1}\right)\frac{p_2}{\rho_2 g} + \frac{V_2^2}{2g} + z_2$$

Assuming $z_1 = z_2$, we have

$$\left(\frac{\gamma}{\gamma-1}\right)\frac{p_1}{\rho_1 g} + \frac{V_1^2}{2g} = \left(\frac{\gamma}{\gamma-1}\right)\frac{p_2}{\rho_2 g} + \frac{V_2^2}{2g}$$

Cancelling '*g*' on both the sides, and rearranging, we get

$$\left(\frac{\gamma}{\gamma-1}\right)\frac{p_1}{\rho_1}\left(1 - \frac{p_2}{p_1} \times \frac{\rho_1}{\rho_2}\right) = \frac{V_2^2}{2} - \frac{V_1^2}{2} \quad ...(i)$$

For an isentropic flow : $\frac{p_1}{\rho_1^\gamma} = \frac{p_2}{\rho_2^\gamma}$ or $\frac{p_1}{p_2} = \left(\frac{\rho_1}{\rho_2}\right)^\gamma$ or $\frac{\rho_1}{\rho_2} = \left(\frac{p_1}{p_2}\right)^{\frac{1}{\gamma}}$

Substituting the value of $\frac{\rho_1}{\rho_2}$ in eqn. (*i*), we have

$$\left(\frac{\gamma}{\gamma-1}\right)\frac{p_1}{\rho_1}\left\{1 - \frac{p_2}{p_1} \times \left(\frac{p_1}{p_2}\right)^{\frac{1}{\gamma}}\right\} = \frac{V_2^2}{2} - \frac{V_1^2}{2}$$

$$\left(\frac{\gamma}{\gamma-1}\right)\frac{p_1}{\rho_1}\left\{1 - \left(\frac{p_2}{p_1}\right)^{1-\frac{1}{\gamma}}\right\} = \frac{V_2^2}{2} - \frac{V_1^2}{2}$$

$$\left(\frac{\gamma}{\gamma-1}\right)\frac{p_1}{\rho_1}\left\{1-\left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}}\right\} = \frac{V_2^2}{2}-\frac{V_1^2}{2}$$

Substituting the values, we get

$$\frac{1.4}{1.4-1}\times 122670\left\{1-\left(\frac{p_2}{p_1}\right)^{\frac{1.4-1}{1.4}}\right\} = \frac{150^2}{2}-\frac{30^2}{2} = 10800$$

$$\left(\because \frac{p_1}{\rho_1} = RT_1 = 290\times 423 = 122670\right)$$

$$429345\left\{1-\left(\frac{p_2}{p_1}\right)^{0.2857}\right\} = 10800$$

or $$\left(\frac{p_2}{p_1}\right)^{0.2857} = 1-\frac{10800}{429345} = 0.9748$$

or $$\frac{p_2}{p_1} = (0.9748)^{1/0.2857} = (0.9748)^{3.5} = 0.9145$$

or $p_2 = 35 \times 0.9145 =$ **32 kN/m²** **(Ans.)**

Temperature, T_2 :

For an isentropic process, we have

$$\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}} = (0.9145)^{\frac{1.4-1}{1.4}} = (0.9145)^{0.2857} = 0.9748$$

$\therefore$ $T_2 = 423 \times 0.9748 = 412.3$ K

or $t_2 = 412.3 - 273 =$ **139.3°C** **(Ans.)**

16.3. PROPAGATION OF DISTURBANCES IN FLUID AND VELOCITY OF SOUND

The solids as well as fluids consist of molecules. Whereas the molecules in solids are close together, these are relatively apart in fluids. Consequently whenever there is a minor disturbance, it travels instantaneously in case of solids ; but in case of fluid the molecules change its position before the transmission or propagation of the disturbance. Thus the velocity of disturbance in case of fluids will be less than the velocity of the disturbance in solids. In case of fluid, the propagation of disturbance depends upon its *elastic properties. The velocity of disturbance depends upon the changes in pressure and density of the fluid.*

The propagation of disturbance is similar to the propagation of sound through a media. The *speed of propagation of sound in a media* is known as *acoustic* or *sonic velocity* and depends upon the difference of pressure. In compressible flow, velocity of sound (sonic velocity) is of paramount importance.

16.3.1. Derivation of Sonic Velocity (velocity of sound)

Consider a one-dimensional flow through long straight rigid pipe of uniform cross-sectional area filled with a frictionless piston at one end as shown in Fig. 16.1. The tube is filled with a compressible fluid initially at rest. If the piston is moved suddenly to the right with a velocity, a *pressure wave* would be propagated through the fluid with the velocity of sound wave.

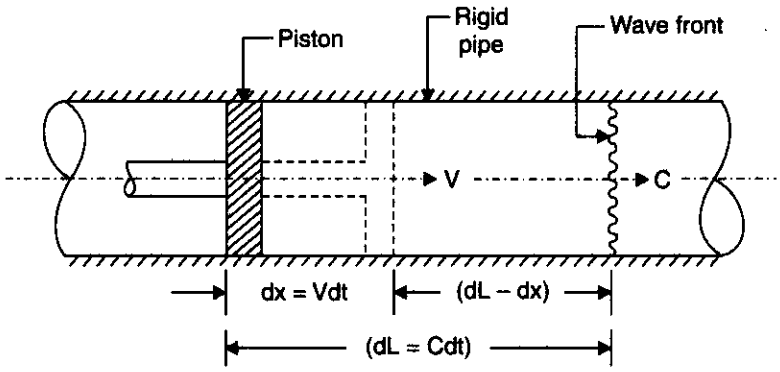


Fig. 16.1. One dimensional pressure wave propagation.

Let A = cross-sectional area of the pipe,

V = piston velocity,

p = fluid pressure in the pipe before the piston movement,

ρ = fluid density before the piston movement,

dt = a small interval of time during which piston moves, and

C = velocity of pressure wave or sound wave (travelling in the fluid).

Before the movement of the piston the length dL has an initial density ρ, and its total mass $= \rho \times dL \times A$.

When the piston moves through a distance dx, the fluid density within the compressed region of length $(dL - dx)$ will be increased and becomes $(\rho + d\rho)$ and subsequently the total mass of fluid in the compressed region $= (\rho + d\rho)(dL - dx) \times A$

$\therefore \quad \rho \times dL \times A = (\rho + d\rho)(dL - dx) \times A$ ...by principle of continuity.

But $dL = C\,dt$ and $dx = Vdt$; therefore, the above equation becomes

$$\rho C dt = (\rho + d\rho)(C - V)\,dt$$

or, $$\rho C = (\rho + d\rho)(C - V) \text{ or } \rho C = \rho C - \rho V + d\rho \,.\, C - d\rho \,.\, V$$

or, $$0 = -\rho V + d\rho \,.\, C - d\rho \,.\, V$$

Neglecting the term $d\rho.V$ (V being much smaller than C), we get

$$d\rho \,.\, C = \rho V \text{ or } C = \frac{\rho V}{d\rho} \quad ...(16.8)$$

Further in the region of compressed fluid, the fluid particles have attained a velocity which is apparently equal to V (velocity of the piston), accompanied by an increase in pressure dp due to sudden motion of the piston. Applying inpulse-momentum equation for the fluid in the compressed region during dt, we get

$$dp \times A \times dt = \rho \times dL \times A\,(V - 0)$$

(force on the fluid) (rate of change of momentum)

or, $$dp = \rho \frac{dL}{dt} V = \rho \times \frac{Cdt}{dt} \times V = \rho CV \quad (\because dL = Cdt)$$

or, $$C = \frac{dp}{\rho V} \quad ...(16.9)$$

Multiplying eqns. (16.8) and (16.9), we get

$$C^2 = \frac{\rho V}{d\rho} \times \frac{dp}{\rho V} = \frac{dp}{d\rho}$$

$$\therefore \qquad C = \sqrt{\frac{dp}{d\rho}} \qquad ...(16.10)$$

16.3.2. Sonic Velocity in Terms of Bulk Modulus

The bulk modulus of elasticity of fluid (*K*) is defined as

$$K = \frac{dp}{\left(\frac{dv}{v}\right)} \qquad ...(i)$$

where, dv = decrease in volume, and v = original volume

(– ve sign indicates that volume decreases with increase in pressure)

Also, volume $v \propto \frac{1}{\rho}$, or $v\rho$ = constant

Differentiating both sides, we get

$$v d\rho + \rho dv = 0 \quad \text{or} \quad -\frac{dv}{v} = \frac{d\rho}{\rho}$$

Substituting the value of $-\frac{dv}{v}\left(=\frac{dp}{K}\right)$ from eqn. (*i*), we get

$$\frac{dp}{K} = \frac{d\rho}{\rho} \quad \text{or} \quad \frac{dp}{d\rho} = \frac{K}{\rho}$$

Substituting this value of $\frac{dp}{d\rho}$ in eqn. (16.10), we get

$$C = \sqrt{\frac{K}{\rho}} \qquad ...(16.11)$$

Eqn. (16.11) is applicable for liquids and gases.

16.3.3. Sonic Velocity for Isothermal Process

For isothermal process : $\frac{p}{\rho}$ = constant

Differentiating both sides, we get

$$\frac{\rho \, . \, dp - p \, . \, d\rho}{\rho^2} = 0 \quad \text{or} \quad \frac{dp}{\rho} - \frac{p \, . \, d\rho}{\rho^2} = 0$$

or,

$$\frac{dp}{\rho} = \frac{p \, . \, d\rho}{\rho^2} \quad \text{or} \quad \frac{dp}{d\rho} = \frac{p}{\rho} = RT \qquad ...(16.12)$$

$$\left(\frac{p}{\rho} = RT \quad \text{...equation of state}\right)$$

Substituting the value of $\frac{dp}{d\rho}$ in eqn. (16.10), we get

$$C = \sqrt{\frac{p}{\rho}} = \sqrt{RT} \qquad ...(16.13)$$

16.3.4. Sonic Velocity for Adiabatic Process

For isentropic (reversible adiabatic) process : $\frac{p}{\rho^{\gamma}}$ = constant

or $p \cdot \rho^{-\gamma}$ = constant

Differentiating both sides, we have $p(-\gamma) \cdot \rho^{-\gamma-1}\, d\rho + \rho^{-\gamma}\, dp = 0$

Dividing both sides by $\rho^{-\gamma}$, we get

$$-p\,\gamma\,\rho^{-1}\, d\rho + dp = 0 \text{ or } dp = p\,\gamma\,\rho^{-1}\, d\rho$$

or, $$\frac{dp}{d\rho} = \frac{p}{\rho}\,\gamma = \gamma RT \qquad \left(\because \frac{p}{\rho} = RT\right)$$

Substituting the value of $\frac{dp}{d\rho}$ in eqn. (16.10), we get

$$C = \sqrt{\gamma RT} \qquad ...(16.14)$$

The following points are worth noting :

(*i*) The process is assumed to be *adiabatic* when minor disturbances are to be propagated through air ; due to *very high velocity* of disturbances/pressure waves appreciable heat transfer does not take place.

(*ii*) For calculation of velocity of the sound/pressure waves, *isothermal process* is considered only when it is mentioned in the numerical problem (that the process is isothermal). When no process is mentioned in the problem, calculation are made assuming the process to be *adiabatic.*

16.4. MACH NUMBER

The mach number is an important parameter in dealing with the flow of compressible fluids, when elastic forces become important and predominant.

Mach number is defined as the *square root of the ratio of the inertia force of a fluid to the elastic force.*

$\therefore$ Mach number, $$M = \sqrt{\frac{\text{Inertia force}}{\text{Elastic force}}} = \sqrt{\frac{\rho A V^2}{KA}}$$

$$= \sqrt{\frac{V^2}{K/\rho}} = \frac{V}{\sqrt{K/\rho}} = \frac{V}{C} \qquad \left[\because \sqrt{K/\rho} = C \text{ ...Eqn. (16.11)}\right]$$

i.e. $$M = \frac{V}{C} \qquad ...(16.15)$$

Thus, $$M = \frac{\text{Velocity at a point in a fluid}}{\text{Velocity of sound at that point at a given instant of time}}$$

Depending on the value of Mach number, the flow can be classified as follows :

1. *Subsonic flow :* Mach number is *less* than 1.0 (or $M < 1$) ; in this case $V < C$.
2. *Sonic flow :* Mach number is *equal* to 1.0 (or $M = 1$) ; in this case $V = C$.
3. *Supersonic flow :* Mach number is *greater* than 1.0 (or $M > 1$) ; in this case $V > C$.

When the Mach number in flow region is slightly less to slightly greater than 1.0, the flow is termed as *transonic flow.*

The following points are worth noting :

(*i*) Mach number is important in those problems in which the flow velocity is comparable with the sonic velocity (velocity of sound). It may happen in case of airplanes travelling at very high speed, projectiles, bullets etc.

(*ii*) If for any flow system the Mach number is less than about 0.4, the effects of compressibility may be neglected (for that flow system).

Example 16.3. *Find the sonic velocity for the following fluids :*

(*i*) *Crude oil of specific gravity 0.8 and bulk modulus 1.5 GN/m^2 ;*

(*ii*) *Mercury having a bulk modulus of 27 GN/m^2.*

(Delhi University)

Sol. Crude oil : Specific gravity = 0.8

∴ Density of oil, $\rho = 0.8 \times 1000 = 800$ kg/m^3

Bulk modulus, $K = 1.5$ GN/m^2

Mercury : Bulk modulus, $K = 27$ GN/m^2

Density of mercury, $\rho = 13.6 \times 1000 = 13600$ kg/m^3

Sonic velocity, C_{oil}, C_{Hg} :

Sonic velocity is given by the relation :

$$C = \sqrt{\frac{K}{\rho}} \quad \text{[Eqn. (16.11)]}$$

$$\therefore \quad C_{oil} = \sqrt{\frac{1.5 \times 10^9}{800}} = \mathbf{1369.3\ m/s} \quad \textbf{(Ans.)}$$

$$C_{Hg} = \sqrt{\frac{27 \times 10^9}{13600}} = \mathbf{1409\ m/s} \quad \textbf{(Ans.)}$$

Example 16.4. *An aeroplane is flying at a height of 14 km where temperature is – 45°C. The speed of the plane is corresponding to M = 2. Find the speed of the plane if R = 287 J/kg K and γ = 1.4.*

Sol. Temperature (at a height of 14 km), $t = -45$°C.

$T = -45 + 273 = 228$ K

Mach number, $M = 2$

Gas constant, $R = 287$ J/kg K

$\gamma = 1.4$

Speed of the plane, V :

Sonic velocity, (*C*) is given by,

$$C = \sqrt{\gamma RT} \quad \text{(assuming the process to be } \textit{adiabatic}\text{)} \quad \text{...[Eqn. (16.14)]}$$

$$= \sqrt{1.4 \times 287 \times 228} = 302.67 \text{ m/s}$$

Also $$M = \frac{V}{C} \quad \text{...[Eqn. (16.15)]}$$

or, $$2 = \frac{V}{302.67}$$

or, $$V = 2 \times 302.67 = 605.34 \text{ m/s} = \frac{605.34 \times 3600}{1000} = \mathbf{2179.2\ km/h} \quad \textbf{(Ans.)}$$

16.5. PROPAGATION OF DISTURBANCE IN COMPRESSIBLE FLUID

When some disturbance is created in a compressible fluid (elastic or pressure waves are also generated), it is propagated in all directions with sonic velocity (= *C*) and its nature of propagation depends upon the Mach number (*M*). Such disturbance may be created when an object moves in a relatively stationary compressible fluid or when a compressible fluid flows past a stationary object.

Consider a tiny projectile moving in a straight line with velocity V through a compressible fluid which is stationary. Let the projectile is at A when time $t = 0$, then in time t it will move through a distance $AB = Vt$. During this time the disturbance which originated from the projectile when it was at A will grow into the surface of sphere of radius Ct as shown in Fig. 16.2, which also shows the growth of the other disturbances which will originate from the projectile at every $t/4$ interval of time as the projectile moves from A to B.

Let us find nature of propagation of the disturbance for different Mach numbers.

Case I. When $M < 1$ (*i.e.,* $V < C$). In this case since $V < C$ the projectile lags behind the disturbance/pressure wave and hence as shown in Fig. 16.2 (*a*) the projectile at point B lies inside the sphere of radius Ct and also inside other spheres formed by the disturbances/waves started at intermediate points.

Case II. When $M = 1$ (*i.e.,* $V = C$). In this case, the disturbance always travels with the projectile as shown in Fig. 16.2 (*b*). The circle drawn with centre A will pass through B.

Case III. When $M > 1$ (*i.e.,* $V > C$). In this case the projectile travels faster than the disturbance. Thus the distance AB (which the projectile has travelled) is more than Ct, and hence

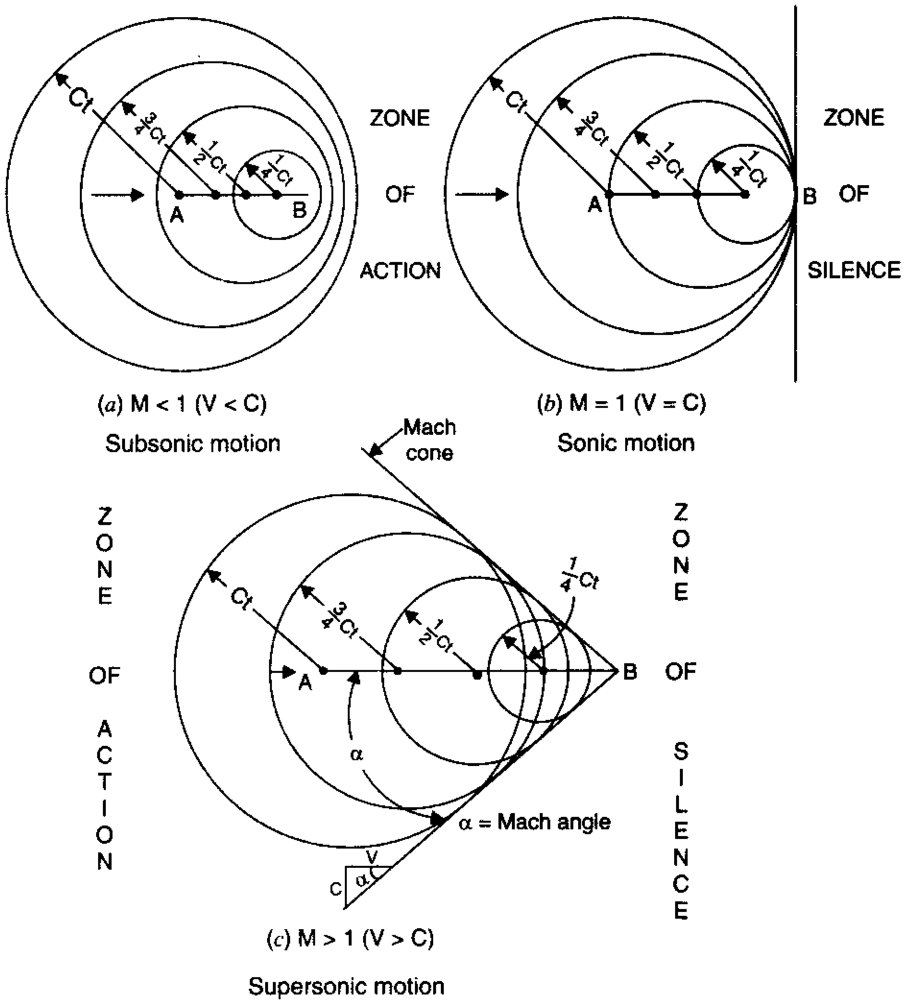


Fig. 16.2. Nature of propagation of disturbances in compressible flow.

the projectile at point 'B' is outside the spheres formed due to formation and growth of disturbance at $t = 0$ and at the intermediate points (Fig. 16.2 (c)). If the tangents are drawn (from the point B) to the circles, the spherical pressure waves form a cone with its vertex at B. It is known as **Mach cone.** The semi-vertex angle α of the cone is known as **Mach angle** which is given by,

$$\sin \alpha = \frac{Ct}{Vt} = \frac{C}{V} = \frac{1}{M} \qquad \text{...(16.16)}$$

In such a case ($M > 1$), the effect of the disturbance is felt only in region inside the Mach cone, this region is called *zone of action*. The region outside the Mach cone is called *zone of silence*.

It has been observed that when an aeroplane is moving with supersonic speed, its noise is heard only after the plane has already passed over us.

Example 16.5. *Find the velocity of a bullet fired in standard air if its Mach angle is 40°.*

Sol. Mach angle, $\alpha = 40°$

$\gamma = 1.4$

For standard air : R = 287 J/kg K, t = 15°C or T = 15 + 273 = 288 K

Velocity of the bullet, V :

Sonic velocity, $C = \sqrt{\gamma RT} = \sqrt{1.4 \times 287 \times 228} = 340.2$ m/s

Now, $\sin \alpha = \frac{C}{V}$

or, $\sin 40° = \frac{340.2}{V}$ or $V = \frac{340.2}{\sin 40°}$ = **529.26 m/s (Ans.)**

Example 16.6. *A projectile is travelling in air having pressure and temperature as 88.3 kN/m² and – 2°C. If the Mach angle is 40°, find the velocity of the projectile. Take γ = 1.4 and R = 287 J/kg K.* **[M.U.]**

Sol. Pressure, p = 88.3 kN/m²

Temperature, T = – 2 + 273 = 271 K

Mach angle, M = 40°

γ = 1.4, R = 287 J/kg K

Velocity of the projectile, V :

Sonic velocity, $C = \sqrt{\gamma RT} = \sqrt{1.4 \times 287 \times 271} \simeq 330$ m/s

Now, $\sin \alpha = \frac{C}{V}$ or $\sin 40° = \frac{340}{V}$

or, $V = \frac{330}{\sin 40°}$ = **513.4 m/s (Ans.)**

Example 16.7. *A supersonic aircraft flies at an altitude of 1.8 km where temperature is 4°C. Determine the speed of the aircraft if its sound is heard 4 seconds after its passage over the head of an observer. Take R = 287 J/kg K and γ = 1.4.*

Sol. Altitude of the aircraft = 1.8 km = 1800 m

Temperature, T = 4 + 273 = 277 K

Time, t = 4 s

Speed of the aircraft, V :

Refer Fig. 16.3. Let O represent the observer and A the position of the aircraft just vertically over the observer. After 4 seconds, the aircraft reaches the position represented by the point B. Line AB represents the wave front and α the Mach angle.

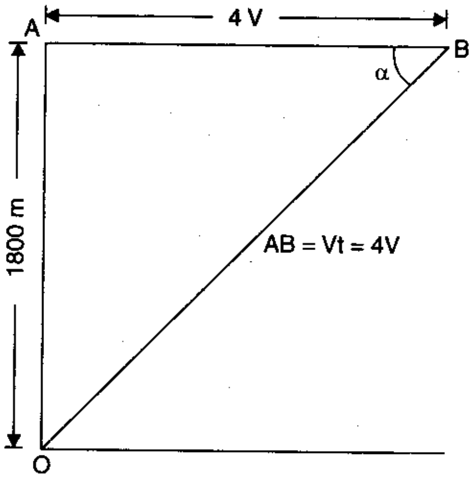


Fig. 16.3

From Fig. 16.3, we have

$$\tan \alpha = \frac{1800}{4V} = \frac{450}{V} \quad ...(i)$$

But, Mach number, $M = \frac{C}{V} = \frac{1}{\sin \alpha}$

or,
$$V = \frac{C}{\sin \alpha} \quad ...(ii)$$

Substituting the value of V in eqn. (i), we get

$$\tan \alpha = \frac{450}{(C/\sin \alpha)} = \frac{450 \sin \alpha}{C}$$

or,
$$\frac{\sin \alpha}{\cos \alpha} = \frac{450 \sin \alpha}{C} \text{ or } \cos \alpha = \frac{C}{450} \quad ...(iii)$$

But $C = \sqrt{\gamma R T}$, where C is the sonic velocity.

$R = 287$ J/kg K and $\gamma = 1.4$ ...(Given)

$\therefore$ $C = \sqrt{1.4 \times 287 \times 277} = 333.6$ m/s

Substituting the value of C in eqn. (ii), we get

$$\cos \alpha = \frac{333.6}{450} = 0.7413$$

$$\therefore \quad \sin \alpha = \sqrt{1 - \cos^2 \alpha} = \sqrt{1 - 0.7413^2} = 0.6712$$

Substituting the value of sin α in eqn. (ii), we get

$$V = \frac{C}{\sin \alpha} = \frac{333.6}{0.6712} = 497 \text{ m/s} = \frac{497 \times 3600}{1000} = \textbf{1789.2 km/h (Ans.)}$$

16.6. STAGNATION PROPERTIES

The point on the immersed body where the velocity is zero is called **stagnation point.** At this point velocity head is converted into pressure head. The values of pressure (p_s), temperature (T_s) and density (ρ_s) at stagnation point are called *stagnation properties.*

16.6.1. Expression for Stagnation Pressure (p_s) in Compressible Flow

Consider the flow of compressible fluid past an immersed body where the velocity becomes zero. Consider *frictionless adiabatic* (*isentropic*) condition. Let us consider two points, O in the free stream and the stagnation point S as shown in Fig. 16.4.

Let, p_0 = pressure of compressible fluid at point O,

V_0 = velocity of fluid at O,

ρ_0 = density of fluid at O,

T_0 = temperature of fluid at O,

and p_s, V_s, ρ_s and T_s are corresponding values of pressure, velocity density, and temperature at point S.

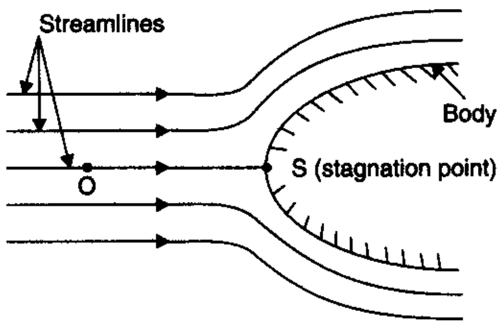


Fig. 16.4. Stagnation properties.

Applying Bernoulli's equation for adiabatic (frictionless) flow at points O and S, (given by eqn. 16.7), we get

$$\left(\frac{\gamma}{\gamma-1}\right)\frac{p_0}{\rho_0 g}+\frac{V_0^2}{2g}+z_0=\left(\frac{\gamma}{\gamma-1}\right)\frac{p_s}{\rho_s g}+\frac{V_s^2}{2g}+z_s$$

But $z_0 = z_s$; the above equation reduces to

$$\left(\frac{\gamma}{\gamma-1}\right)\frac{p_0}{\rho_0 g}+\frac{V_0^2}{2g}=\left(\frac{\gamma}{\gamma-1}\right)\frac{p_s}{\rho_s g}+\frac{V_s^2}{2g}$$

Cancelling 'g' on both the sides, we have

$$\left(\frac{\gamma}{\gamma-1}\right)\frac{p_0}{\rho_0}+\frac{V_0^2}{2}=\left(\frac{\gamma}{\gamma-1}\right)\frac{p_s}{\rho_s}+\frac{V_s^2}{2}$$

At point S the velocity is zero, *i.e.*, $V_s = 0$; the above equation becomes

$$\left(\frac{\gamma}{\gamma-1}\right)\left(\frac{p_0}{\rho_0}-\frac{p_s}{\rho_s}\right)=-\frac{V_0^2}{2}$$

or,
$$\left(\frac{\gamma}{\gamma-1}\right)\frac{p_0}{\rho_0}\left(1-\frac{p_s}{\rho_s}\times\frac{\rho_0}{p_0}\right)=-\frac{V_0^2}{2}$$

or,
$$\left(\frac{\gamma}{\gamma-1}\right)\frac{p_0}{\rho_0}\left(1-\frac{p_s}{p_0}\times\frac{\rho_0}{\rho_s}\right)=-\frac{V_0^2}{2} \quad ...(i)$$

For adiabatic process : $\dfrac{p_0}{\rho_0^\gamma}=\dfrac{p_s}{\rho_s^\gamma}$ or $\dfrac{p_0}{p_s}=\dfrac{\rho_0^\gamma}{\rho_s^\gamma}$ or $\dfrac{\rho_0}{\rho_s}=\left(\dfrac{p_0}{p_s}\right)^{\frac{1}{\gamma}}$ ...(ii)

Substituting the value of $\dfrac{\rho_0}{\rho_s}$ in eqn. (*i*), we get

$$\left(\frac{\gamma}{\gamma-1}\right)\frac{p_0}{\rho_0}\left[1-\frac{p_s}{p_0}\times\left(\frac{p_0}{p_s}\right)^{\frac{1}{\gamma}}\right]=-\frac{V_0^2}{2}$$

or,
$$\left(\frac{\gamma}{\gamma-1}\right)\frac{p_0}{\rho_0}\left\{1-\left(\frac{p_s}{p_0}\right)^{1-\frac{1}{\gamma}}\right\}=-\frac{V_0^2}{2}$$

or,
$$\left[1-\left(\frac{p_s}{p_0}\right)^{\frac{\gamma-1}{\gamma}}\right]=-\frac{V_0^2}{2}\left(\frac{\gamma-1}{\gamma}\right)\frac{\rho_0}{p_0}$$

or,
$$1+\frac{V_0^2}{2}\left(\frac{\gamma-1}{\gamma}\right)\frac{\rho_0}{p_0}=\left(\frac{p_s}{p_0}\right)^{\frac{\gamma-1}{\gamma}} \quad ...(iii)$$

For adiabatic process, the sonic velocity is given by,

$$C = \sqrt{\gamma RT} = \sqrt{\gamma \frac{p}{\rho}} \qquad \left(\because \frac{p}{\rho} = RT\right)$$

For point O,

$$C_0 = \sqrt{\gamma \frac{p_0}{\rho_0}} \text{ or } C_0^2 = \gamma \frac{p_0}{\rho_0}$$

Substituting the value of $\frac{\gamma p_0}{\rho_0} = C_0^2$ in eqn. (*iii*), we get

$$1 + \frac{V_0^2}{2}(\gamma - 1) \times \frac{1}{C_0^2} = \left(\frac{p_s}{p_0}\right)^{\frac{\gamma-1}{\gamma}}$$

or,

$$1 + \frac{V_0^2}{2C_0^2}(\gamma - 1) = \left(\frac{p_s}{p_0}\right)^{\frac{\gamma-1}{\gamma}}$$

$$1 + \frac{M_0^2}{2}(\gamma - 1) = \left(\frac{p_s}{p_0}\right)^{\frac{\gamma-1}{\gamma}} \qquad \left(\because \frac{V_0^2}{C_0^2} = M_0^2\right)$$

or,

$$\left(\frac{p_s}{p_0}\right)^{\frac{\gamma-1}{\gamma}} = \left[1 + \left(\frac{\gamma-1}{2}\right) M_0^2\right]$$

or,

$$\frac{p_s}{p_0} = \left[1 + \left(\frac{\gamma-1}{2}\right) M_0^2\right]^{\frac{\gamma}{\gamma-1}} \qquad \text{...(iv)}$$

or,

$$p_s = p_0 \left[1 + \left(\frac{\gamma-1}{2}\right) M_0^2\right]^{\frac{\gamma}{\gamma-1}} \qquad \text{...(16.17)}$$

Eqn. (16.17) gives the value of **stagnation pressure.**

Compressibility correction factor :

If the right hand side of eqn. (16.17) is expanded by the binomial theorem, we get

$$p_s = p_0 \left[1 + \frac{\gamma}{2} M_0^2 + \frac{\gamma}{8} M_0^4 + \frac{\gamma(2-\gamma)}{48} M_0^6\right]$$

$$= p_0 \left[1 + \frac{\gamma M_0^2}{2}\left(1 + \frac{M^2}{4} + \frac{2-\gamma}{24} M_0^4 + \ldots\right)\right]$$

or,

$$p_s = p_0 + \frac{p_0 \gamma M_0^2}{2}\left(1 + \frac{M_0^2}{4} + \frac{2-\gamma}{24} M_0^4 + \ldots\right) \qquad \text{...(16.18)}$$

But,

$$M_0^2 = \frac{V_0^2}{C_0^2} = \frac{V_0^2}{\left(\frac{\gamma p_0}{\rho_0}\right)} = \frac{V_0^2 \rho_0}{\gamma p_0} \qquad \left(\because C_0^2 = \frac{\gamma p_0}{\rho_0}\right)$$

Substituting the value of M_0^2 in eqn. (16.18), we get

$$p_s = p_0 + \frac{p_0\gamma}{2} \times \frac{V_0^2\rho_0}{\gamma p_0}\left(1 + \frac{M_0^2}{4} + \frac{2-\gamma}{24} M_0^4 + \ldots\right)$$

or,
$$p_s = p_0 + \frac{\rho_0 V_0^2}{2}\left(1 + \frac{M_0^2}{4} + \frac{2-\gamma}{24} M_0^4 + \ldots\right) \quad \ldots(16.19)$$

Also, $p_s = p_0 + \dfrac{\rho_0 V_0^2}{2}$ (when compressibility effects are neglected) ...(16.20)

The comparison of eqns. (16.19) and (16.20) shows that the effects of compressibility are isolated in the bracketed quantity and that these effects *depend only* upon the *Mach number*. The bracketed quantity $\left[i.e., \left(1 + \frac{M_0^2}{4} + \frac{2-\gamma}{24} M_0^4 + \ldots\right)\right]$ may thus be considered as a **compressibility correction factor.** It is worth noting that :

- For $M < 0.2$, the compressibility affects the pressure difference $(p_s - p_0)$ by *less than 1 per cent* and the simple formula for flow at constant density is then sufficiently accurate.
- For larger value of M, as the terms of binomial expansion become significant, the compressibility effect must be taken into account.
- When the Mach number exceeds a value of about 0.3 the *Pitot-static tube used for measuring aircraft speed needs calibration to take into account the compressibility effects.*

16.6.2. Expression for Stagnation Density (ρ_s)

From eqn. (*ii*), we have

$$\frac{\rho_0}{\rho_s} = \left(\frac{p_0}{p_s}\right)^{\frac{1}{\gamma}} \text{ or } \frac{\rho_s}{\rho_0} = \left(\frac{p_s}{p_0}\right)^{\frac{1}{\gamma}} \text{ or } \rho_s = \rho_0\left(\frac{p_s}{p_0}\right)^{\frac{1}{\gamma}}$$

Substituting the value of $\left(\dfrac{p_s}{p_0}\right)$ from eqn. (*iv*), we get

$$\rho_s = \rho_0\left[\left\{1 + \left(\frac{\gamma-1}{2}\right)M_0^2\right\}^{\frac{\gamma}{\gamma-1}}\right]^{\frac{1}{\gamma}}$$

or,
$$\rho_s = \rho_0\left[1 + \left(\frac{\gamma-1}{2}\right)M_0^2\right]^{\frac{1}{\gamma-1}} \quad \ldots(16.21)$$

16.6.3. Expression for Stagnation Temperature (T_s)

The equation of state is given by : $\dfrac{p}{\rho} = RT$

For stagnation point, the equation of state may be written as :

$$\frac{p_s}{\rho_s} = RT_s \text{ or } T_s = \frac{1}{R}\frac{p_s}{\rho_s}$$

Substituting the values of p_s and ρ_s from eqns. (16.17) and (16.18), we get

$$T_s = \frac{1}{R}\frac{p_0\left[1+\left(\frac{\gamma-1}{2}\right)M_0^2\right]^{\frac{\gamma}{\gamma-1}}}{\rho_0\left[1+\left(\frac{\gamma-1}{2}\right)M_0^2\right]^{\frac{1}{\gamma-1}}}$$

$$= \frac{1}{R}\frac{p_0}{\rho_0}\left[1+\left(\frac{\gamma-1}{2}\right)M_0^2\right]^{\left(\frac{\gamma}{\gamma-1}-\frac{1}{\gamma-1}\right)}$$

$$= \frac{1}{R}\frac{p_0}{\rho_0}\left[1+\left(\frac{\gamma-1}{2}\right)M_0^2\right]^{\left(\frac{\gamma-1}{\gamma-1}\right)}$$

or,
$$T_s = T_0\left[1+\left(\frac{\gamma-1}{2}\right)M_0^2\right] \qquad ...(16.22) \qquad \left(\because \frac{p_0}{\rho_0} = RT_0\right)$$

Example 16.8. *An aeroplane is flying at 1000 km/h through still air having a pressure of 78.5 kN/m² (abs.) and temperature – 8°C. Calculate on the stagnation point on the nose of the plane :*

(i) Stagnation pressure, *(ii) Stagnation temperature, and*

(iii) Stagnation density.

Take for air : R = 287 J/kg K and γ = 1.4.

Sol. Speed of aeroplane, $V = 1000 \text{ km/h} = \frac{1000 \times 1000}{60 \times 60} = 277.77 \text{ m/s}$

Pressure of air, $p_0 = 78.5 \text{ kN/m}^2$

Temperature of air, $T_0 = -8 + 273 = 265 \text{ K}$

For air : $R = 287$ J/kg K, $\gamma = 1.4$

The sonic velocity for adiabatic flow is given by,

$$C_0 = \sqrt{\gamma RT_0} = \sqrt{1.4 \times 287 \times 265} = 326.31 \text{ m/s}$$

∴ Mach number, $M_0 = \frac{V_0}{C_0} = \frac{277.77}{326.31} = 0.851$

(*i*) **Stagnation pressure, p_s :**

The stagnation pressure (p_s) is given by the relation,

$$p_s = p_0\left[1+\left(\frac{\gamma-1}{2}\right)M_0^2\right]^{\frac{\gamma}{\gamma-1}} \qquad ...[\text{Eqn. } (16.17)]$$

or,
$$p_s = 78.5\left[1+\left(\frac{1.4-1}{2}\right)\times 0.851^2\right]^{\frac{1.4}{1.4-1}}$$

$$= 78.5\,(1.145)^{3.5} = \mathbf{126.1\ kN/m^2} \quad \textbf{(Ans.)}$$

(*ii*) **Stagnation temperature, T_s :**

The stagnation temperature is given by,

$$T_s = T_0 \left[1 + \left(\frac{\gamma - 1}{2}\right) M_0^2\right] \quad \text{...[Eqn. (16.22)]}$$

$$= 265 \left[1 + \frac{1.4 - 1}{2} \times 0.851^2\right] = 303.4 \text{ K or } \mathbf{30.4°C} \quad \textbf{(Ans.)}$$

(*iii*) **Stagnation density, ρ_s :**

The stagnation density (ρ_s) is given by,

$$\frac{p_s}{\rho_s} = RT_s \quad \text{or} \quad \rho_s = \frac{p_s}{RT_s}$$

or,
$$\rho_s = \frac{126.1 \times 10^3}{287 \times 303.4} = \mathbf{1.448 \ kg/m^3} \quad \textbf{(Ans.)}$$

Example 16.9. *Air has a velocity of 1000 km/h at a pressure of 9.81 kN/m² in vacuum and a temperature of 47°C. Compute its stagnation properties and the local Mach number. Take atmospheric pressure = 98.1 kN/m², R = 287 J/kg K and γ = 1.4.*

What would be the compressibility correction factor for a pitot-static tube to measure the velocity at a Mach number of 0.8.

Sol. Velocity of air, $V_0 = 1000 \text{ km/h} = \frac{1000 \times 1000}{60 \times 60} = 277.78 \text{ m/s}$

Temperature of air, $T_0 = 47 + 273 = 320 \text{ K}$

Atmospheric pressure, $p_{atm} = 98.1 \text{ kN/m}^2$

Pressure of air (static), $p_0 = 98.1 - 9.81 = 88.29 \text{ kN/m}^2$

$R = 287 \text{ J/kg K}, \ \gamma = 1.4$

Sonic velocity, $C_0 = \sqrt{\gamma R T_0} = \sqrt{1.4 \times 287 \times 320} = 358.6 \text{ m/s}$

∴ Mach number, $M_0 = \frac{V_0}{C_0} = \frac{277.78}{358.6} = 0.7746$

Stagnation pressure, p_s :

The stagnation pressure is given by,

$$p_s = p_0 \left[1 + \left(\frac{\gamma - 1}{2}\right) M_0^2\right]^{\frac{\gamma}{\gamma - 1}} \quad \text{...[Eqn. (16.17)]}$$

or,
$$p_s = 88.29 \left[1 + \frac{1.4 - 1}{2} \times 0.7746^2\right]^{\frac{1.4}{1.4 - 1}}$$

$$= 88.29 \ (1.12)^{3.5} = \mathbf{131.27 \ kN/m^2} \quad \textbf{(Ans.)}$$

Stagnation temperature, T_s :

$$T_s = T_0 \left[1 + \left(\frac{\gamma - 1}{2}\right) M_0^2\right] \quad \text{...[Eqn. (16.22)]}$$

or,
$$T_s = 320 \left[1 + \frac{1.4 - 1}{2} \times 0.7746^2\right] = 358.4 \text{ K or } \mathbf{85.4°C} \quad \textbf{(Ans.)}$$

Stagnation density, ρ_s :

$$\rho_s = \frac{p_s}{RT_s} = \frac{131.27 \times 10^3}{287 \times 358.4} = \mathbf{1.276\ kg/m^3} \quad \textbf{(Ans.)}$$

Compressibility factor at M = 0.8 :

Compressibility factor $= 1 + \frac{M_0^2}{4} + \frac{2-\gamma}{24} M_0^4 + ...$

$$= 1 + \frac{0.8^2}{4} + \frac{2-1.4}{24} \times 0.8^4 = \mathbf{1.1702} \quad \textbf{(Ans.)}$$

Example 16.10. *Air at a pressure of 220 kN/m² and temperature 27°C is moving at a velocity of 200 m/s. Calculate the stagnation pressure if*

(i) Compressibility is neglected ; *(ii) Compressibility is accounted for.*

For air take R = 287 J/kg K, γ = 1.4.

Sol. Pressure of air, p_0 = 200 kN/m²

Temperature of air, T_0 = 27 + 233 = 300 K

Velocity of air, V_0 = 200 m/s

Stagnation pressure, p_s :

(*i*) **Compressibility is neglected :**

$$p_s = p_0 + \frac{\rho_0 V_0^2}{2}$$

where $\rho_0 = \frac{p_0}{RT_0} = \frac{220 \times 10^3}{287 \times 300} = 2.555$ kg/m³

$$\therefore \quad p_s = 220 + \frac{2.555 \times 200^2}{2} \times 10^{-3}\ (\text{kN/m}^2) = \mathbf{271.1\ kN/m^2.\ Ans.}$$

(*ii*) **Compressibility is accounted for :**

The stagnation pressure, when compressibility is accounted for, is given by,

$$p_s = p_0 + \frac{\rho_0 V_0^2}{2}\left(1 + \frac{M_0^2}{4} + \frac{2-\gamma}{24} M_0^4 + ...\right) \quad ...[\text{Eqn. (16.19)}]$$

Mach number, $M_0 = \frac{V_0}{C_0} = \frac{200}{\sqrt{\gamma R T_0}} = \frac{200}{\sqrt{1.4 \times 287 \times 300}} = 0.576$

Whence, $p_s = 220 + \frac{2.555 \times 200^2}{2} \times 10^{-3}\left(1 + \frac{0.576^2}{4} + \frac{2-1.4}{24} \times 0.576^4\right)$

or, p_s = 220 + 51.1 (1 + 0.0829 + 0.00275) = **275.47 kN/m² (Ans.)**

Example 16.11. *In aircraft flying at an altitude where the pressure was 35 kPa and temperature – 38°C, stagnation pressure measured was 65.4 kPa. Calculate the speed of the aircraft. Take molecular weight of air as 28.* **(UPSC, 1998)**

Sol. Pressure of air, p_0 = 35 kPa = 35 × 10³ N/m²

Temperature of air, T_0 = – 38 + 273 = 235 K

Stagnation pressure, p_s = 65.4 kPa = 65.4 × 10³ N/m²

Speed of the aircraft, V_a :

$$p_0 V = mRT_0 = m \times \left(\frac{R_0}{M}\right) T_0 \text{ or } \rho_0 = \frac{m}{V} = \frac{p_0 M}{R_0 T_0}$$

where R = characteristic gas constant,

R_0 = universal gas constant = 8314 Nm/mole K.

M = molecular weight for air = 28, and

ρ_0 = density of air.

Substituting the values, we get

$$\rho_0 = \frac{(35 \times 10^3) \times 28}{8314 \times 235} = 0.5 \text{ kg/m}^3$$

Now, using the relation : $p_s = p_0 + \frac{\rho_0 V_a^2}{2}$ ...[Eqn. (16.20)]

or, $$V_a = \sqrt{\frac{2(p_s - p)}{\rho_0}} = \sqrt{\frac{2(65.4 \times 10^3 - 35 \times 10^3)}{0.5}} = \textbf{348.7 m/s} \quad \textbf{(Ans.)}$$

16.7. AREA-VELOCITY RELATIONSHIP AND EFFECT OF VARIATION OF AREA FOR SUBSONIC, SONIC AND SUPERSONIC FLOWS

For an incompressible flow the continuity equation may be expressed as :

AV = constant, which when differentiated gives

$$AdV + VdA = 0 \quad \text{or} \quad \frac{dA}{A} = -\frac{dV}{V} \qquad ...(16.23)$$

But in case of compressible flow, the continuity equation is given by,

ρAV = constant, which can be differentiated to give

$$\rho d(AV) + AVd\rho = 0 \quad \text{or} \quad \rho(AdV + VdA) + AVd\rho = 0$$

or, $$\rho AdV + \rho VdA + AVd\rho = 0$$

Dividing both sides by ρAV, we get

$$\frac{dV}{V} + \frac{dA}{A} + \frac{d\rho}{\rho} = 0 \qquad ...(16.24)$$

or, $$\frac{dA}{A} = -\frac{dV}{V} - \frac{d\rho}{\rho} \qquad ...[16.24\ (a)]$$

The Euler's equation for compressible fluid is given by,

$$\frac{dp}{\rho} + VdV + gdz = 0$$

Neglecting the z terms the above equation reduces to, $\frac{dp}{\rho} + VdV = 0$

This equation can also be expressed as :

$$\frac{dp}{\rho} \times \frac{d\rho}{d\rho} + VdV = 0 \quad \text{or} \quad \frac{dp}{d\rho} \times \frac{d\rho}{\rho} + VdV = 0$$

But $$\frac{dp}{d\rho} = C^2 \qquad ...[\text{Eqn. (16.10)}]$$

$$\therefore \quad C^2 \times \frac{d\rho}{\rho} + VdV = 0 \quad \text{or} \quad C^2 \frac{d\rho}{\rho} = -VdV \quad \text{or} \quad \frac{d\rho}{\rho} = -\frac{VdV}{C^2}$$

Substituting the value of $\frac{d\rho}{\rho}$ in eqn. (16.24), we get

$$\frac{dV}{V} + \frac{dA}{A} - \frac{VdV}{C^2} = 0$$

or,
$$\frac{dA}{A} = \frac{VdV}{C^2} - \frac{dV}{V} = \frac{dV}{V}\left(\frac{V^2}{C^2} - 1\right)$$

$\therefore$
$$\frac{dA}{A} = \frac{dV}{V}(M^2 - 1) \quad \left(\because M = \frac{V}{C}\right) \quad ...(16.25)$$

This important equation is due to *Hugoniot*.

Eqns. (16.23) and (16.25) give variation of $\left(\frac{dA}{A}\right)$ for the flow of incompressible and compressible fluids respectively. The ratios $\left(\frac{dA}{A}\right)$ and $\left(\frac{dV}{V}\right)$ are respectively fractional variations in the values of area and flow velocity in the flow passage.

Further, in order to study the variation of pressure with the change in flow area, an expression similar to eqn. (16.25), as given below, can be obtained.

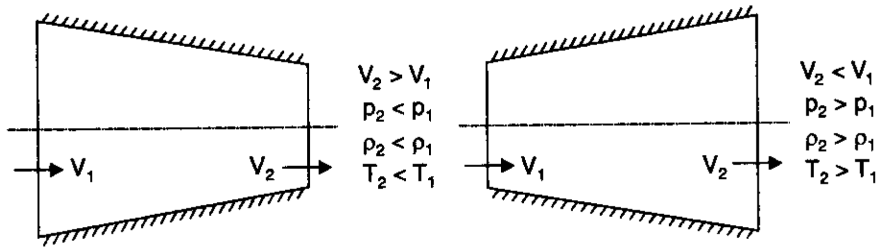
$$dp = \rho V^2 \left(\frac{1}{1 - M^2}\right)\frac{dA}{A} \quad ...(16.26)$$

From eqns. (16.25) and (16.26), it is possible to formulate the following conclusions of practical significance.

(*i*) **For subsonic flow (M < 1) :**

$$\frac{dV}{V} > 0 \; ; \; \frac{dA}{A} < 0 \; ; \; dp < 0 \text{ (convergent nozzle)}$$

$$\frac{dV}{V} < 0 \; ; \; \frac{dA}{A} > 0 \; ; \; dp > 0 \text{ (divergent diffuser)}$$



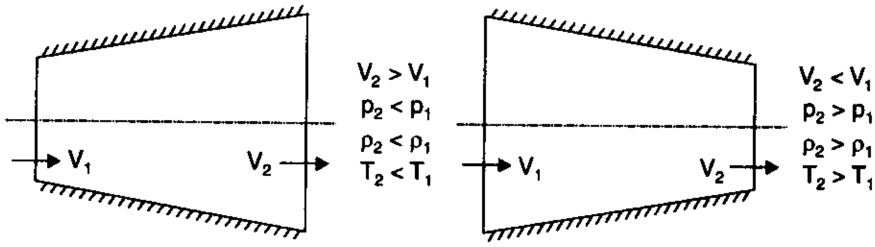
(*a*) Convergent nozzle. (*b*) Divergent diffuser.

Fig. 16.5. Subsonic flow ($M < 1$).

(*ii*) **For supersonic flow (M > 1) :**

$$\frac{dV}{V} > 0 \; ; \; \frac{dA}{A} > 0 \; ; \; dp < 0 \text{ (divergent nozzle)}$$

$$\frac{dV}{V} < 0 \; ; \; \frac{dA}{A} < 0 \; ; \; dp > 0 \text{ (convergent diffuser)}$$



(*a*) Divergent nozzle. (*b*) Convergent nozzle.

Fig. 16.6. Supersonic flow ($M > 1$).

(*iii*) **For sonic flow (M = 1) :**

$$\frac{dA}{A} = 0 \text{ (straight flow passage}$$

since dA must be zero)

and dp = (zero/zero) *i.e.*, indeterminate, but when evaluated, the change of pressure $p = 0$, since $dA = 0$ and the flow is frictionless.

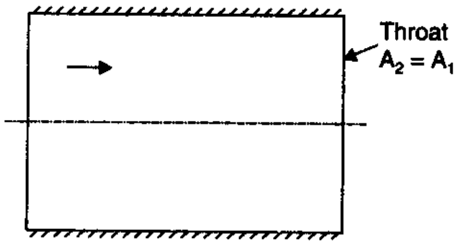


Fig. 16.7. Sonic flow ($M = 1$).

16.8. FLOW OF COMPRESSIBLE FLUID THROUGH A CONVERGENT NOZZLE

Fig. 16.8 shows a large tank/vessel fitted with a short convergent nozzle and containing a compressible fluid. Consider two points 1 and 2 inside the tank and exit of the nozzle respectively.

Let p_1 = pressure of fluid at the point 1,

V_1 = velocity of fluid in the tank (= 0),

T_1 = temperature of fluid at point 1,

ρ_1 = density of fluid at point 1, and p_2, V_2, T_2 and ρ_2 are corresponding values of pressure, velocity, temperature and density at point 2.

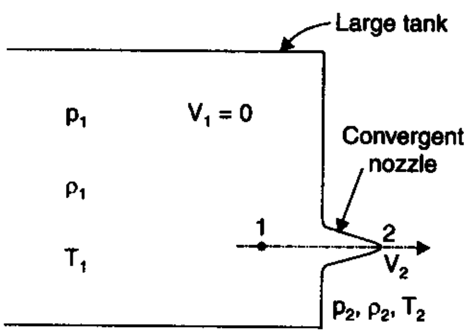


Fig. 16.8. Flow of fluid through a convergent nozzle.

Assuming the flow to take place *adiabatically*, then by using Bernoulli's equation (for adiabatic flow), we have

$$\left(\frac{\gamma}{\gamma-1}\right)\frac{p_1}{\rho_1 g} + \frac{V_1^2}{2g} + z_1 = \left(\frac{\gamma}{\gamma-1}\right)\frac{p_2}{\rho_2 g} + \frac{V_2^2}{2g} + z_2 \qquad \text{[Eqn. (16.7)]}$$

But $z_1 = z_2$ and $V_1 = 0$

$$\therefore \qquad \frac{\gamma}{\gamma-1}\frac{p_1}{\rho_1 g} = \left(\frac{\gamma}{\gamma-1}\right)\frac{p_2}{\rho_2 g} + \frac{V_2^2}{2g}$$

or,

$$\left(\frac{\gamma}{\gamma-1}\right)\left[\frac{p_1}{\rho_1 g} - \frac{p_2}{\rho_2 g}\right] = \frac{V_2^2}{2g} \quad \text{or} \quad \frac{\gamma}{\gamma-1}\left[\frac{p_1}{\rho_1} - \frac{p_2}{\rho_2}\right] \frac{V_2^2}{2}$$

or
$$V_2 = \sqrt{\frac{2\gamma}{(\gamma-1)}\left(\frac{p_1}{\rho_1} - \frac{p_2}{\rho_2}\right)}$$

or
$$V_2 = \sqrt{\frac{2\gamma}{(\gamma-1)}\frac{p_1}{\rho_1}\left(1 - \frac{p_2}{\rho_2} \cdot \frac{\rho_1}{p_1}\right)} \quad ...(1)$$

For adiabatic flow : $\dfrac{p_1}{\rho_1^{\gamma}} = \dfrac{p_2}{\rho_2^{\gamma}}$ or $\dfrac{p_1}{p_2} = \left(\dfrac{\rho_1}{\rho_2}\right)^{\gamma}$ or $\dfrac{\rho_1}{\rho_2} = \left(\dfrac{p_1}{p_2}\right)^{\frac{1}{\gamma}}$ ...(*i*)

Substituting the value of $\dfrac{\rho_1}{\rho_2}$ in eqn. (1), we get

$$V_2 = \sqrt{\frac{2\gamma}{(\gamma-1)}\frac{p_1}{\rho_1}\left[1 - \frac{p_2}{p_1} \times \left(\frac{p_1}{p_2}\right)^{\frac{1}{\gamma}}\right]} = \sqrt{\frac{2\gamma}{(\gamma-1)}\frac{p_1}{\rho_1}\left[1 - \left(\frac{p_2}{p_1}\right)^{1-\frac{1}{\gamma}}\right]}$$

or
$$V_2 = \sqrt{\frac{2\gamma}{(\gamma-1)}\frac{p_1}{\rho_1}\left[1 - \left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}}\right]} \quad ...(16.27)$$

The mass rate of flow of the compressible fluid,

$m = \rho_2 A_2 V_2$, A_2 being the area of the nozzle at the exit

$$= \rho_2 A_2 \sqrt{\frac{2\gamma}{(\gamma-1)}\frac{p_1}{\rho_1}\left[1 - \left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}}\right]}, \quad \text{[substituting } V_2 \text{ from eqn. (16.27)]}$$

or
$$m = A_2 \sqrt{\frac{2\gamma}{(\gamma-1)}\frac{p_1}{\rho_1} \times \rho_2^2 \left[1 - \left(\frac{p_2}{\rho_1}\right)^{\frac{\gamma-1}{\gamma}}\right]}$$

From eqn. (*i*), we have $\rho_2 = \dfrac{\rho_1}{(p_1/p_2)^{1/\gamma}} = \rho_1 \left(\dfrac{p_2}{p_1}\right)^{\frac{1}{\gamma}}$

$\therefore$
$$\rho_2^2 = \rho_1^2 \left(\frac{p_2}{p_1}\right)^{\frac{2}{\gamma}}$$

Substituting this value in the above equation, we get

$$m = A_2 \sqrt{\frac{2\gamma}{\gamma-1}\frac{p_1}{\rho_1} \times \rho_1^2 \left(\frac{p_2}{p_1}\right)^{\frac{2}{\gamma}} \left[1 - \left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}}\right]}$$

$$= A_2 \sqrt{\frac{2\gamma}{\gamma-1} p_1 \rho_1 \left[\left(\frac{p_2}{p_1}\right)^{2/\gamma} - \left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}+\frac{2}{\gamma}}\right]}$$

$$m = A_2 \sqrt{\frac{2\gamma}{\gamma - 1} p_1 \rho_1 \left[\left(\frac{p_2}{p_1}\right)^{2/\gamma} - \left(\frac{p_2}{p_1}\right)^{\frac{\gamma+1}{\gamma}} \right]} \quad ...(16.28)$$

The mass rate of flow (m) depends on the value of $\frac{p_2}{p_1}$ (for the given values of p_1 and ρ_1 at point 1).

Value of $\frac{p_2}{p_1}$ for maximum value of mass rate of flow :

For maximum value of m, we have $\frac{d}{d\left(\frac{p_2}{p_1}\right)}(m) = 0$

As other quantities except the ratio $\frac{p_2}{p_1}$ are constant

$$\therefore \quad \frac{d}{d\left(\frac{p_2}{p_1}\right)}(m) = \left[\left(\frac{p_2}{p_1}\right)^{2/\gamma} - \left(\frac{p_2}{p_1}\right)^{\frac{\gamma+1}{\gamma}} \right] = 0$$

or,
$$\frac{2}{\gamma}\left(\frac{p_2}{p_1}\right)^{\frac{2}{\gamma}-1} - \left(\frac{\gamma+1}{\gamma}\right)\left(\frac{p_2}{p_1}\right)^{\frac{\gamma+1}{\gamma}-1} = 0$$

or,
$$\left(\frac{p_2}{p_1}\right)^{\frac{2}{\gamma}-1} = \left(\frac{\gamma+1}{2}\right)\left(\frac{p_2}{p_1}\right)^{\frac{1}{\gamma}} \text{ or } \left(\frac{p_2}{p_1}\right)^{\frac{2-\gamma}{\gamma}} = \frac{\gamma+1}{2}\left(\frac{p_2}{p_1}\right)^{\frac{1}{\gamma}}$$

or,
$$\left(\frac{p_2}{p_1}\right)^{2-\gamma} = \left(\frac{\gamma+1}{2}\right)^{\gamma}\left(\frac{p_2}{p_1}\right)$$

or,
$$\left(\frac{p_2}{p_1}\right)^{2-\gamma-1} = \left(\frac{\gamma+1}{2}\right)^{\gamma} \text{ or } \left(\frac{p_2}{p_1}\right)^{1-\gamma} = \left(\frac{\gamma+1}{2}\right)^{\gamma}$$

or,
$$\left(\frac{p_2}{p_1}\right)^{\gamma-1} = \left(\frac{2}{\gamma+1}\right)^{\gamma}$$

or,
$$\left(\frac{p_2}{p_1}\right) = \left(\frac{2}{\gamma+1}\right)^{\frac{\gamma}{\gamma-1}} \quad ...(16.29)$$

Eqn. (16.29) is the *condition for maximum mass flow rate through the nozzle.*

It may be pointed out that a convergent nozzle is employed when the exit pressure is equal to or more than the critical pressure, and a convergent-divergent nozzle is used when the discharge pressure is less than the critical pressure.

For air with $\gamma = 1.4$, the *critical pressure ratio,*

$$\frac{p_2}{p_1} = \left(\frac{2}{1.4+1}\right)^{\frac{1.4}{1.4-1}} = 0.528 \quad ...(16.30)$$

Relevant relations for critical density and temperature are :

$$\frac{\rho_2}{\rho_1} = \left(\frac{2}{\gamma+1}\right)^{\frac{1}{\gamma-1}} \qquad \text{...[16.30 (a)]}$$

$$\frac{T_2}{T_1} = \frac{2}{\gamma+1} \qquad \text{...[16.30 (b)]}$$

Value of V_2 for maximum rate of flow of fluid :

Substituting the value of $\frac{p_2}{p_1}$ from eqn. (16.29) in eqn. (16.27), we get

$$V_2 = \sqrt{\frac{2\gamma}{\gamma-1}\frac{p_1}{\rho_1}\left[1-\left(\frac{2}{\gamma+1}\right)^{\frac{\gamma}{\gamma-1}\times\frac{\gamma-1}{\gamma}}\right]} = \sqrt{\frac{2\gamma}{\gamma-1}\frac{p_1}{\rho_1}\left(1-\frac{2}{\gamma+1}\right)}$$

$$= \sqrt{\frac{2\gamma}{\gamma-1}\frac{p_1}{\rho_1}\left(\frac{\gamma+1-2}{\gamma+1}\right)} = \sqrt{\frac{2\gamma}{\gamma-1}\frac{p_1}{\rho_1}\left(\frac{\gamma-1}{\gamma+1}\right)}$$

or $$V_2 = \sqrt{\frac{2\gamma}{\gamma+1}\frac{p_1}{\rho_1}} \;(= C_2) \qquad \text{...(16.31)}$$

Maximum rate of flow of fluid through nozzle, m_{max} :

Substituting the value of $\frac{p_2}{p_1}$ from eqn. (16.30) in eqn. (16.28), we get

$$m_{max} = A_2\sqrt{\left(\frac{2\gamma}{\gamma-1}\right)p_1\rho_1\left[\left(\frac{2}{\gamma+1}\right)^{\frac{\gamma}{\gamma+1}\times\frac{2}{\gamma}} - \left(\frac{2}{\gamma+1}\right)^{\frac{\gamma}{\gamma-1}\times\frac{\gamma+1}{\gamma}}\right]}$$

$$= A_2\sqrt{\left(\frac{2\gamma}{\gamma-1}\right)p_1\rho_1\left[\left(\frac{2}{\gamma+1}\right)^{\frac{2}{\gamma-1}} - \left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{\gamma-1}}\right]}$$

For air, $\gamma = 1.4$,

$$\therefore \quad m_{max} = A_2\sqrt{\frac{2\times1.4}{(1.4-1)}\,p_1\rho_1\left[\left(\frac{2}{1.4+1}\right)^{\frac{2}{1.4-1}} - \left(\frac{2}{1.4+1}\right)^{\frac{1.4+1}{1.4-1}}\right]}$$

$$= A_2\sqrt{7\,p_1\rho_1\,(0.4018-0.3348)}$$

or $$m_{max} = 0.685\,A_2\sqrt{p_1\rho_1} \qquad \text{...(16.32)}$$

Variation of mass flow rate of compressible fluid with pressure ratio $\left(\frac{p_2}{p_1}\right)$:

A passage in which the sonic velocity has been reached and thus in which the mass flow rate is maximum, is often said to be **choked** or in **choking conditions**. It is evident from eqn. (16.28) that for a fixed value of inlet pressure the mass flow depends on nozzle exit pressure.

Fig. 16.9. depicts the variation of actual and theoretical mass flow rate versus $\frac{p_2}{p_1}$. Following points are *worthnoting* :

(*i*) The flow rate increases with a decrease in the pressure ratio $\frac{p_2}{p_1}$ and attains the maximum value of the critical pressure ratio $\frac{p_2}{p_1} = 0.528$ for air.

(*ii*) With further decrease in exit pressure below the critical value, the theoretical mass flow rate decreases. This is contrary to the actual results where the mass flow rate remains constant after attaining the maximum value. This may be explained as follows :

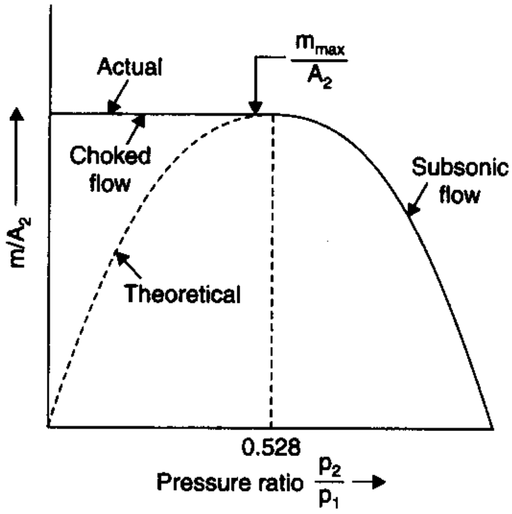


Fig. 16.9. Mass flow rate through a convergent nozzle.

At critical pressure ratio, the velocity V_2 at the throat is equal to the sonic speed (derived below). For an accelerating flow of a compressible fluid in a convergent nozzle the velocity of flow within the nozzle is subsonic with a maximum velocity equal to the sonic velocity at the throat : Thus once the velocity V_2 at the throat has attained the sonic speed at the critical pressure ratio, it remains at the same value for all the values of $\left(\frac{p_2}{p_1}\right)$ less than critical pressure ratio, since the flow in the nozzle is being continuously accelerated with the reduction in the throat pressure below the critical values and hence the velocity cannot reduce. Thus, the mass flow rate for all values of $\left(\frac{p_2}{p_1}\right)$ less than critical pressure ratio remains constant at the maximum value (indicated by the solid horizontal line in Fig. 16.9). This fact has been verified experimentally too.

Velocity at outlet of nozzle for maximum flow rate :

The velocity at outlet of nozzle for maximum flow rate is given by,

$$V_2 = \sqrt{\left(\frac{2\gamma}{\gamma+1}\right)\frac{p_1}{\rho_1}} \qquad \text{...[Eqn. (16.31)]}$$

Now pressure ratio,
$$\frac{p_2}{p_1} = \left(\frac{2}{\gamma+1}\right)^{\frac{\gamma}{\gamma-1}}$$

$\therefore$
$$p_1 = \frac{p_2}{\left(\frac{2}{\gamma+1}\right)^{\frac{\gamma}{\gamma-1}}} = p_2\left(\frac{2}{\gamma+1}\right)^{\frac{\gamma}{\gamma-1}}$$

For *adiabatic flow* :
$$\frac{p_1}{\rho_1^{\gamma}} = \frac{p_2}{\rho_2^{\gamma}} \quad \text{or} \quad \frac{p_1}{p_2} = \left(\frac{\rho_1}{\rho_2}\right)^{\gamma} \quad \text{or} \quad \frac{\rho_1}{\rho_2} = \left(\frac{p_1}{p_2}\right)^{\frac{1}{\gamma}} = \left(\frac{p_2}{p_1}\right)^{-\frac{1}{\gamma}}$$

$$\therefore \quad \rho_1 = \rho_2 \left(\frac{p_2}{p_1}\right)^{-\frac{1}{\gamma}} \text{ or } \rho_2 \left(\frac{2}{\gamma+1}\right)^{\frac{\gamma}{\gamma-1}\times\frac{1}{\gamma}} \text{ or } \rho_2 \left(\frac{2}{\gamma+1}\right)^{\frac{1}{1-\gamma}}$$

Substituting the values of p_1 and ρ_1 in the above eqn. (16.31), we get

$$V_2 = \sqrt{\left(\frac{2\gamma}{\gamma+1}\right) \times p_2 \left(\frac{2}{\gamma+1}\right)^{\frac{\gamma}{1-\gamma}} \times \left\{\frac{1}{\rho_2} \times \left(\frac{2}{\gamma+1}\right)^{\frac{1}{\gamma-1}}\right\}}$$

$$= \sqrt{\left(\frac{2\gamma}{\gamma+1}\right) \times \frac{p_2}{\rho_2} \times \left(\frac{2}{\gamma+1}\right)^{\frac{\gamma}{1-\gamma}+\frac{1}{\gamma-1}}} = \sqrt{\left(\frac{2\gamma}{\gamma+1}\right) \times \frac{p_2}{\rho_2} \left(\frac{2}{\gamma+1}\right)^{-1}}$$

or $$V_2 = \sqrt{\left(\frac{2\gamma}{\gamma+1}\right) \times \frac{p_2}{\rho_2} \left(\frac{\gamma+1}{2}\right)} = \sqrt{\frac{\gamma p_2}{\rho_2}} = C_2$$

i.e., $$V_2 = C_2 \qquad \text{...(16.33)}$$

Hence the *velocity* at the outlet of nozzle for maximum flow rate equals sonic velocity.

16.9. VARIABLES OF FLOW IN TERMS OF MACH NUMBER

In order to obtain relationship involving change in velocity, pressure, temperature and density in terms of the Mach number use is made of the continuity, perfect gas, isentropic flow and energy equations.

For *continuity equation*, we have

$$\rho AV = \text{constant}$$

Differentiating the above equation, we get

$$\rho[AdV + VdA] + AVd\rho = 0$$

Dividing throughout by ρAV, we have

$$\frac{dV}{V} + \frac{dA}{A} + \frac{d\rho}{\rho} = 0$$

From *isentropic flow*, we have $\frac{p}{\rho^\gamma}$ = constant or $\frac{dp}{p} = \gamma \frac{dp}{p}$

For *perfect gas*, we have $p = \rho RT$ or $\frac{dp}{p} = \frac{d\rho}{\rho} + \frac{dT}{T}$

From *energy equation*, we have $c_pT + \frac{V^2}{2}$ + constant

Differentiating throughout, we get

$$c_p dT + VdV = 0 \text{ or } \left(\frac{\gamma R}{\gamma-1}\right) dT + VdV = 0 \qquad \left(\because c_p = \frac{\gamma R}{\gamma-1}\right)$$

or, $$\frac{\gamma R}{\gamma-1} \frac{dT}{V^2} + \frac{dV}{V} = 0 \qquad \text{...}(i)$$

Also, sonic velocity, $C = \sqrt{\gamma RT}$ $\quad \therefore \gamma R = \frac{C^2}{T}$

Substituting the value of $\gamma R = \frac{C^2}{T}$ in eqn. (*i*), we get

$$\frac{C^2}{(\gamma - 1)T} \times \frac{dT}{V^2} + \frac{dV}{V} = 0$$

or, $$\frac{1}{(\gamma - 1) M^2} \times \frac{dT}{T} + \frac{dV}{V} = 0 \qquad \left(\because \; M = \frac{V}{C}\right) \qquad ...(16.34)$$

From the Mach number relationship

$$M = \frac{V}{\sqrt{\gamma RT}} \text{ (where } \sqrt{\gamma RT} = C\text{)}$$

$$\frac{dM}{M} = \frac{dV}{V} - \frac{1}{2}\frac{dT}{T} \qquad ...(16.35)$$

Substituting the value of $\frac{dT}{T}$ from eqns. (16.34) in eqn. (16.35), we get

$$\frac{dM}{M} = \frac{dV}{V} - \frac{1}{2}\left[-\frac{dV}{V} \times (\gamma - 1) M^2\right]$$

$$= \frac{dV}{V} + \frac{1}{2}\frac{dV}{V} \times (\gamma - 1) M^2$$

or, $$\frac{dM}{M} = \frac{dV}{V}\left[1 + \frac{\gamma - 1}{2} M^2\right] \text{ or } \frac{dV}{V} = \frac{1}{\left[1 + \left(\frac{\gamma - 1}{2}\right) M^2\right]}\frac{dM}{M} \qquad ...(16.36)$$

Since the quantity within the bracket is *always positive*, the *trend of variation of velocity and Mach number is similar. For temperature variation, one can write*

$$\frac{dT}{T} = \left[\frac{-(\gamma - 1) M^2}{1 + \left(\frac{\gamma - 1}{2}\right) M^2}\right]\frac{dM}{M} \qquad ...(16.37)$$

Since the right hand side is *negative the temperature changes follow an opposite trend to that of Mach number*. Similarly for *pressure and density*, we have

$$\frac{dp}{p} = \left[\frac{-\gamma M^2}{1 + \frac{\gamma - 1}{2} M^2}\right]\frac{dM}{M} \qquad ...(16.38)$$

and, $$\frac{d\rho}{\rho} = \left[\frac{-M^2}{1 + \left(\frac{\gamma - 1}{2}\right) M^2}\right]\frac{dM}{M} \qquad ...(16.39)$$

For changes in *area*, we have

$$\frac{dA}{A} = \left[\frac{-(1 - M^2)}{1 + \frac{\gamma - 1}{2} M^2}\right]\frac{dM}{M} \qquad ...(16.40)$$

The quantity within the brackets may be *positive or negative* depending upon the *magnitude of Mach number*. By integrating eqn. (16.40), we can obtain a relationship between the critical throat area A_c, where Mach number is *unity* and the area A at any section where $M \gtrless 1$

$$\frac{A}{A_c} = \frac{1}{M}\left[\frac{2+(\gamma-1)M^2}{\gamma+1}\right]^{\frac{\gamma+1}{2(\gamma-1)}} \quad ...(16.41)$$

Example 16.12. *The pressure leads from Pitot-static tube mounted on an aircraft were connected to a pressure gauge in the cockpit. The dial of the pressure gauge is calibrated to read the aircraft speed in m/s. The calibration is done on the ground by applying a known pressure across the gauge and calculating the equivalent velocity using incompressible Bernoulli's equation and assuming that the density is 1.224 kg/m³.*

The gauge having been calibrated in this way the aircraft is flown at 9200 m, where the density is 0.454 kg/m³ and ambient pressure is 30 kN/m². The gauge indicates a velocity of 152 m/s. What is the true speed of the aircraft ? **(UPSC)**

Sol. Bernoulli's equation for an *incompressible flow* is given by,

$$p + \frac{\rho V^2}{2} = \text{constant}$$

The stagnation pressure (p_s) created at Pitot-static tube,

$$p_s = p_0 + \frac{\rho_0 V_0^2}{2} \text{ (neglecting compressibility effects)} \quad ...(i)$$

Here $p_0 = 30$ kN/m², $V_0 = 152$ m/s, $\rho_0 = 1.224$ kg/m³ ...(*Given*)

$$\therefore \quad p_s = 30 + \frac{1.224 \times 152^2}{2} \times 10^{-3} = 44.139 \text{ kN/m}^2$$

Neglecting compressibility effect, the speed of the aircraft when $\rho_0 = 0.454$ kg/m³ is given by [using eqn. (*i*)],

$$44.139 \times 10^3 = 30 \times 10^3 + \frac{0.454 \times V_0^2}{2}$$

or

$$V_0^2 = \frac{(44.139 - 30) \times 10^3 \times 2}{0.454} = 62286.34$$

$$\therefore \quad V_0 = 249.57 \text{ m/s}$$

Sonic velocity, $$C_0 = \sqrt{\gamma R T_0} = \sqrt{\gamma \frac{p_0}{\rho_0}} = \sqrt{1.4 \times \frac{30 \times 10^3}{0.454}} = 304.16 \text{ m/s}$$

Mach number, $$M = \frac{V_0}{C_0} = \frac{249.57}{304.16} = 0.82$$

Compressibility correction factor $= \left(1 + \frac{M_0^2}{4}\right)$, neglecting the terms containing higher powers of M_0 (from eqn 16.19).

$$= \left(1 + \frac{0.82}{4}\right) = 1.168$$

$\therefore$ True speed of aircraft $= \dfrac{249.57}{\sqrt{1168}} = 230.9$ m/s

Hence true speed of aircraft = **230.9 m/s (Ans.)**

Example 16.13. (*a*) *In case of isentropic flow of a compressible fluid through a variable duct, show that*

$$\frac{A}{A_c} = \frac{1}{M}\left[\frac{1+\frac{1}{2}(\gamma-1)M^2}{\frac{1}{2}(\gamma+1)}\right]^{\frac{\gamma+1}{2(\gamma-1)}}$$

where γ is the ratio of specific heats, M is the Mach number at a section whose area is A and A_c is the critical area of flow.

(*b*) *A supersonic nozzle is to be designed for air flow with Mach number 3 at the exit section which is 200 mm in diameter. The pressure and temperature of air at the nozzle exit are to be 7.85 kN/m² and 200 K respectively. Determine the reservoir pressure, temperature and the throat area. Take : γ = 1.4.* **(U.P.S.C. Exam.)**

Sol. (*a*) Please Ref. to Art. 16.9.

(*b*) Mach number, $M = 3$

Area at the exit section, $A = \pi/4 \times 0.2^2 = 0.0314 \text{ m}^2$

Pressure of air at the nozzle, $(p)_{\text{nozzle}} = 7.85 \text{ kN/m}^2$

Temperature of air at the nozzle, $(T)_{\text{nozzle}} = 200 \text{ K}$

Reservoir pressure, $(p)_{res.}$:

From eqn. (16.17), $$(p)_{res.} = (p)_{\text{nozzle}}\left[1+\left(\frac{\gamma-1}{2}\right)M^2\right]^{\left(\frac{\gamma}{\gamma-1}\right)}$$

or, $$(p)_{res.} = 7.85\left[1+\left(\frac{1.4-1}{2}\right)\times 3^2\right]^{\left(\frac{1.4}{1.4-1}\right)} = \textbf{288.35 kN/m}^2 \quad \textbf{(Ans.)}$$

Reservoir temperature, $(T)_{res.}$:

From eqn. (16.22), $$(T)_{res.} = (T)_{\text{nozzle}}\left[1+\left(\frac{\gamma-1}{2}\right)M^2\right]$$

or, $$(T)_{res.} = 200\left[1+\left(\frac{1.4-1}{2}\right)\times 3^2\right] = \textbf{560 K} \quad \textbf{(Ans.)}$$

Throat area (critical), A_c :

From eqn. (16.41), $$\frac{A}{A_c} = \frac{1}{M}\left[\frac{2+(\gamma-1)M^2}{\gamma+1}\right]^{\frac{\gamma+1}{2(\gamma-1)}}$$

or, $$\frac{0.0314}{A_c} = \frac{1}{3}\left[\frac{2+(1.4-1)3^2}{1.4+1}\right]^{\frac{1.4+1}{2(1.4-1)}} \quad \text{or} \quad \frac{0.0314}{A_c} = \frac{1}{3}(2.333)^3 = 4.23$$

or, $$A_c = \frac{0.0314}{4.23} = \textbf{0.00742 m}^2 \quad \textbf{(Ans.)}$$

16.10. FLOW THROUGH LAVAL NOZZLE (CONVERGENT-DIVERGENT NOZZLE)

Laval nozzle is a convergent-divergent nozzle (named after de Laval, the swedish scientist who invented it) in which *subsonic flow prevails in the converging section, critical or transonic conditions in the throat* and *supersonic flow in the diverging section.*

Let p_2 (= p_c) = pressure in the throat when the flow is sonic for given pressure p_1.

— When the pressure in the receiver, $p_3 = p_1$, there will be no flow through the nozzle, this is shown by line a in Fig. 16.10 (b).

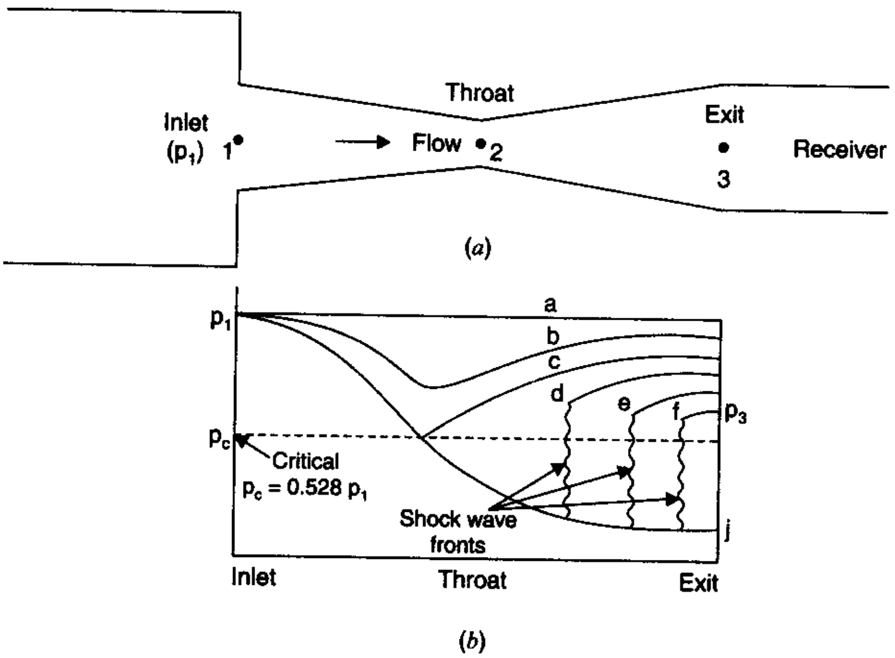


Fig. 16.10. (a) Laval nozzle (convergent-divergent nozzle) ; (b) Pressure distribution through a convergent-divergent nozzle with flow of compressible fluid.

— When the receiver pressure is reduced, flow will occur through the nozzle. As long as the value of p_3 is such that throat pressure p_2 is greater than the critical pressure 0.528 p_1, the flow in the converging and diverging sections will be subsonic. This condition is shown by line 'b'.

— With further reduction in p_3, a stage is reached when p_2 is equal to critical pressure $p_c = 0.528\ p_1$, at this line $M = 1$ in the throat. This condition is shown by line 'c'. Flow is subsonic on the upstream as well the downstream of the throat. The flow is also *isentropic*.

— *If p_3 is further reduced, it does not effect the flow in convergent section. The flow in throat is sonic, downstream it is supersonic.* Somewhere in the diverging section a shock wave occurs and flow changes to subsonic (curve d). The flow *across the shock is non-isentropic. Downstream of the shock wave the flow is subsonic and decelerates.*

— If the value of p_3 is further reduced, the shock wave forms somewhat downstream (curve e).

— For p_3 equal to p_j, the shock wave will occur just at the exit of divergent section.

— If the value of p_3 lies before p_f and p_j oblique waves are formed at the exit.

Example 16.14. *A large tank contains air at 284 kN/m² gauge pressure and 24°C temperature. The air flows from the tank to the atmosphere through a convergent nozzle. If the diameter at the outlet of the nozzle is 20 mm, find the maximum flow rate of air.*

Take : $R = 287$ J/kg K, $\gamma = 1.4$ and atmospheric pressure = 100 kN/m².

(Punjab University)

Sol. Pressure in the tank, $p_1 = 284$ kN/m^2 (gauge)

$= 284 + 100 = 384$ kN/m^2 (absolute)

Temperature in the tank, $T_1 = 24 + 273 = 297$ K

Diameter at the outlet of the nozzle, $D = 20$ mm $= 0.02$ m

$$\therefore \quad \text{Area, } A_2 = \frac{\pi}{4} \times 0.02^2 = 0.0003141 \text{ m}^2$$

$$R = 287 \text{ J/kg K}, \gamma = 1.4$$

(Two points are considered. Point 1 lies inside the tank and point 2 lies at the exit of the nozzle).

Maximum flow rate, m_{max} :

Equation of state is given by $p = \rho RT$ or $\rho = \frac{p}{RT}$

$$\therefore \quad \rho_1 = \frac{p_1}{RT_1} = \frac{384 \times 10^3}{287 \times 297} = 4.5 \text{ kg/m}^3$$

The fluid parameters in the tank correspond to the stagnation values, and maximum flow rate of air is given by,

$$m_{max} = 0.685 \, A_2 \sqrt{p_1 \rho_1} \qquad \text{...[Eqn. (16.32)]}$$

$$= 0.685 \times 0.0003141 \sqrt{384 \times 10^3 \times 4.5} = 0.283 \text{ kg/s}$$

Hence maximum flow rate of air = **0.283 kg/s (Ans.)**

Example 16.15. *A large vessel, fitted with a nozzle, contains air at a pressure of 2500 kN/m^2 (abs.) and at a temperature of 20°C. If the pressure at the outlet of the nozzle is 1750 kN/m^2, find the velocity of air flowing at the outlet of the nozzle.*

Take : $R = 287$ J/kg K and $\gamma = 1.4$.

Sol. Pressure inside the vessel, $p_1 = 2500$ kN/m^2 (abs.)

Temperature inside the vessel, $T_1 = 20 + 273 = 293$ K

Pressure at the outlet of the nozzle, $p_2 = 1750$ kN/m^2 (abs.)

$$R = 287 \text{ J/kg K}, \gamma = 1.4$$

Velocity of air, V_2 :

$$V_2 = \sqrt{\left(\frac{2\gamma}{\gamma - 1}\right) \frac{p_1}{\rho_1} \left[1 - \left(\frac{p_2}{p_1}\right)^{\frac{\gamma - 1}{\gamma}}\right]} \qquad \text{...[Eqn. (16.27)]}$$

where,

$$\rho_1 = \frac{p_1}{RT_1} \left(\text{From equation of state} : \frac{p}{\rho} = RT\right)$$

$$= \frac{2500 \times 10^3}{287 \times 293} = 29.73 \text{ kg/m}^3$$

Substituting the values in the above equation, we get

$$V_2 = \sqrt{\left(\frac{2 \times 1.4}{1.4 - 1}\right) \times \frac{2500 \times 10^3}{29.73} \left[1 - \left(\frac{1750}{2500}\right)^{\frac{1.4 - 1}{1.4}}\right]}$$

$$= \sqrt{7 \times 84090\,(1 - 0.903)} = 238.9 \text{ m/s}$$

i.e., $V_2 =$ **238.9 m/s (Ans.)**

Example 16.16. *A tank fitted with a convergent nozzle contains air at a temperature of 20°C. The diameter at the outlet of the nozzle is 25 mm. Assuming adiabatic flow, find the mass rate of flow of air through the nozzle to the atmosphere when the pressure in the tank is :*

(i) 140 kN/m² (abs.), *(ii) 300 kN/m²*

Take for air : R = 287 J/kg K and γ = 1.4, Barometric pressure = 100 kN/m².

Sol. Temperature of air in the tank, $T_1 = 20 + 273 = 293$ K

Diameter at the outlet of the nozzle, $D_2 = 25$ mm $= 0.025$ m

Area, $A_2 = \pi/4 \times 0.025^2 = 0.0004908 \text{ m}^2$

$R = 287$ J/kg K, $\gamma = 1.4$

(*i*) **Mass rate of flow of air when pressure in the tank is 140 kN/m² (abs.) :**

$$\rho_1 = \frac{p_1}{RT_1} = \frac{140 \times 10^3}{287 \times 293} = 1.665 \text{ kg/m}^3$$

$$p_1 = 140 \text{ kN/m}^2 \text{ (abs.)}$$

Pressure at the nozzle, p_2 = atmospheric pressure = 100 kN/m²

∴ Pressure ratio, $\dfrac{p_2}{p_1} = \dfrac{100}{140} = 0.7143$

Since the pressure ratio is more than the critical value, flow in the nozzle will be *subsonic*, hence mass rate of flow of air is given by eqn. 16.28, as

$$m = A_2 \sqrt{\frac{2\gamma}{\gamma - 1} p_1 \rho_1 \left[\left(\frac{p_2}{p_1}\right)^{\frac{2}{\gamma}} - \left(\frac{p_2}{p_1}\right)^{\frac{\gamma+1}{\gamma}} \right]}$$

$$= 0.0004908 \sqrt{\left(\frac{2 \times 1.4}{1.4 - 1}\right) \times 140 \times 10^3 \times 1.665 \left[(0.7143)^{\frac{2}{1.4}} - (0.7143)^{\frac{1.4+1}{1.4}} \right]}$$

$$= 0.0004908 \sqrt{1631700\,(0.7143)^{1.4285} - (0.7143)^{1.7142}}$$

or $$m = 0.0004908 \sqrt{1631700\,(0.6184 - 0.5617)} = \mathbf{0.1493 \text{ kg/s}} \quad \textbf{(Ans.)}$$

(*ii*) **Mass rate of flow of air when pressure in the tank is 300 kN/m² (abs.) :**

$p_1 = 300$ kN/m² (abs.)

p_2 = pressure at the nozzle = atmospheric pressure = 100 kN/m²

∴ Pressure ratio, $\dfrac{p_2}{p_1} = \dfrac{100}{300} = 0.33.$

The pressure ratio being less than the critical ratio 0.528, the flow in the nozzle will be *sonic*, the flow rate is maximum and is given by eqn. (16.32), as

$$m_{max} = 0.685\, A_2 \sqrt{p_1 \rho_1}$$

where, $$\rho_1 = \frac{p_1}{RT_1} = \frac{300 \times 10^3}{287 \times 293} = 3.567 \text{ kg/m}^3$$

∴ $$m_{max} = 0.685 \times 0.0004908 \sqrt{300 \times 10^3 \times 3.567} = \mathbf{0.3477 \text{ kg/s}} \quad \textbf{(Ans.)}$$

Example 16.17. *At some section in the convergent-divergent nozzle, in which air is flowing, pressure, velocity, temperature and cross-sectional area are 200 kN/m², 170 m/s, 200°C and 1000 mm² respectively. If the flow conditions are isentropic, determine :*

(*i*) *Stagnation temperature and stagnation pressure,*

(*ii*) *Sonic velocity and Mach number at this section,*

(*iii*) *Velocity, Mach number and flow area at outlet section where pressure is 110 kN/m²,*

(*iv*) *Pressure, temperature, velocity and flow area at throat of the nozzle.*

Take for air : $R = 287$ *J/kg K,* $c_p = 1.0$ *kJ/kg K* and $\gamma = 1.4$.

Sol. Let subscripts 1, 2 and t refers to the conditions at given section, outlet section and throat section of the nozzle respectively.

Pressure in the nozzle, $p_1 = 200$ kN/m²

Velocity of air, $V_1 = 170$ m/s

Temperature, $T_1 = 200 + 273 = 473$ K

Cross-sectional area, $A_1 = 1000 \text{ mm}^2 = 1000 \times 10^{-6} = 0.001 \text{ m}^2$

For air : $R = 287$ J/kg K, $c_p = 1.0$ kJ/kg K, $\gamma = 1.4$

(*i*) **Stagnation temperature (T_s) and stagnation pressure (p_s) :**

Stagnation temperature, $$T_s = T_1 + \frac{V_1^2}{2 \times c_p}$$

$$= 473 + \frac{170^2}{2 \times (1.0 \times 1000)} = \textbf{487.45 K (or 214.45°C)} \quad \textbf{(Ans.)}$$

Also, $$\frac{p_s}{p_1} = \left(\frac{T_s}{T_1}\right)^{\frac{\gamma}{\gamma-1}} = \left(\frac{487.45}{473}\right)^{\frac{1.4}{1.4-1}} = 1.111$$

∴ Stagnation pressure, $p_s = 200 \times 1.111 = \textbf{222.2 kN/m}^2$ **(Ans.)**

(*ii*) **Sonic velocity and Mach number at this section :**

Sonic velocity, $$C_1 = \sqrt{\gamma R T_1} = \sqrt{1.4 \times 287 \times 473} = \textbf{435.9 m/s} \quad \textbf{(Ans.)}$$

Mach number, $$M_1 = \frac{V_1}{C_1} = \frac{170}{435.9} = \textbf{0.39} \quad \textbf{(Ans.)}$$

(*iii*) **Velocity, Mach number and flow area at outlet section where pressure is 110 kN/m² :**

Pressure at outlet section, $p_2 = 110$ kN/m² ...(*Given*)

From eqn (16.17), $$\frac{p_s}{p_1} = \left[1 + \left(\frac{\gamma - 1}{2}\right) M_2^2\right]^{\frac{\gamma}{\gamma-1}}$$

$$\frac{222.2}{110} = \left[1 + \left(\frac{1.4 - 1}{2}\right) M_0^2\right]^{\frac{1.4}{1.4-1}} = (1 + 0.2\, M_0^2)^{3.5}$$

or, $$(1 + 0.2\, M_2^2) = \left(\frac{222.2}{110}\right)^{\frac{1}{3.5}} = 1.222$$

or, $$M_2 = \left(\frac{1.222 - 1}{0.2}\right)^{1/2} = \textbf{1.05} \quad \textbf{(Ans.)}$$

Also, $$\frac{T_2}{T_s} = \left(\frac{p_2}{p_s}\right)^{\frac{\gamma-1}{\gamma}} = \left(\frac{110}{222.2}\right)^{\frac{1.4-1}{1.4}} = 0.818$$

or, $$T_2 = 0.818 \times 487.45 = 398.7 \text{ K}$$

Sonic velocity at outlet section,

$$C_2 = \sqrt{\gamma R T_2} = \sqrt{1.4 \times 287 \times 398.7} = 400.25 \text{ m/s}$$

∴ Velocity at outlet section, $V_2 = M_2 \times C_2 = 1.05 \times 400.25 =$ **420.26 m/s. Ans.**

Now, mass flow at the given section = mass flow at outlet section (exit)

......continuity equation

i.e., $$\rho_1 A_1 V_1 = \rho_2 A_2 V_2 \quad \text{or} \quad \frac{p_1}{RT_1} A_1 V_1 = \frac{p_2}{RT_2} A_2 V_2$$

∴ Flow area at the outlet section,

$$A_2 = \frac{p_1 A_1 V_1 T_2}{T_1 p_2 V_2} = \frac{200 \times 0.001 \times 170 \times 398.7}{473 \times 110 \times 420.26} = 6.199 \times 10^{-4} \text{ m}^2$$

Hence, $A_2 = 6.199 \times 10^{-4}$ m² or **619.9 mm². Ans.**

(*iv*) **Pressure (p_t), temperature (T_t), velocity (V_t), and flow area (A_t) at throat of the nozzle :**

At throat, *critical conditions prevail, i.e.* the flow velocity becomes *equal to the sonic velocity* and *Mach number attains* a unit value.

From eqn. (16.22), $$\frac{T_s}{T_t} = \left[1 + \left(\frac{\gamma - 1}{2}\right) M_t^2\right]$$

or, $$\frac{487.45}{T_t} = \left[1 + \left(\frac{1.4-1}{2}\right) \times 1^2\right] = 1.2 \text{ or } T_t = 406.2 \text{ K}$$

Hence T_t = **406.2 K (or 133.2°C). Ans.**

Also, $$\frac{p_t}{T_s} = \left(\frac{T_t}{T_s}\right)^{\frac{\gamma}{\gamma-1}} \text{ or } \frac{p_t}{222.2} = \left(\frac{406.2}{487.45}\right)^{\frac{1.4}{1.4-1}} = 0.528$$

or, $p_t = 222.2 \times 0.528 =$ **117.32 kN/m². Ans.**

Sonic velocity (corresponding to throat conditions),

$$C_t = \sqrt{\gamma R T_t} = \sqrt{1.4 \times 287 \times 406.2} = 404 \text{ m/s}$$

∴ Flow velocity, $V_t = M_t \times C_t = 1 \times 404 = 404$ m/s

By continuity equation, we have : $\rho_1 A_1 V_1 = \rho_t A_t V_t$

or, $$\frac{p_1}{RT_1} A_1 V_1 = \frac{p_t}{RT_t} A_t V_t$$

∴ Flow area at throat, $$A_t = \frac{p_1 A_1 V_1 T_t}{T_1 p_t V_t} = \frac{200 \times 0.001 \times 170 \times 406.2}{473 \times 117.32 \times 404} = 6.16 \times 10^{-4} \text{ m}^2$$

Hence, $A_t = 6.16 \times 10^{-4}$ m² or **616 mm² (Ans.)**

16.11. SHOCK WAVES

Whenever a supersonic flow (compressible) abruptly changes to subsonic flow, a shock *wave* (analogous to hydraulic jump in an open channel) is produced, *resulting in a sudden rise in pressure, density, temperature and entropy*. This occurs due to pressure differentials and when the Mach number of the approaching flow $M_1 > 1$. A shock wave is a pressure wave of finite thickness, of the order of 10^{-2} to 10^{-4} mm in the atmospheric pressure. A shock wave takes place in the *diverging section of a nozzle*, in a *diffuser, throat of a supersonic wind tunnel*, in *front of sharp nosed bodies*.

Shock waves are of two types :

1. Normal shocks which are almost perpendicular to the flow.
2. Oblique shocks which are inclined to the flow direction.

16.11.1. Normal Shock Wave

Consider a duct having a compressible sonic flow (see Fig. 16.11).

Let p_1, ρ_1, T_1, and V_1 be the pressure, density, temperature and velocity of the flow ($M_1 > 1$) and p_2, ρ_2, T_2 and V_2 the corresponding values of pressure, density, temperature and velocity after a shock wave takes place ($M_2 < 1$).

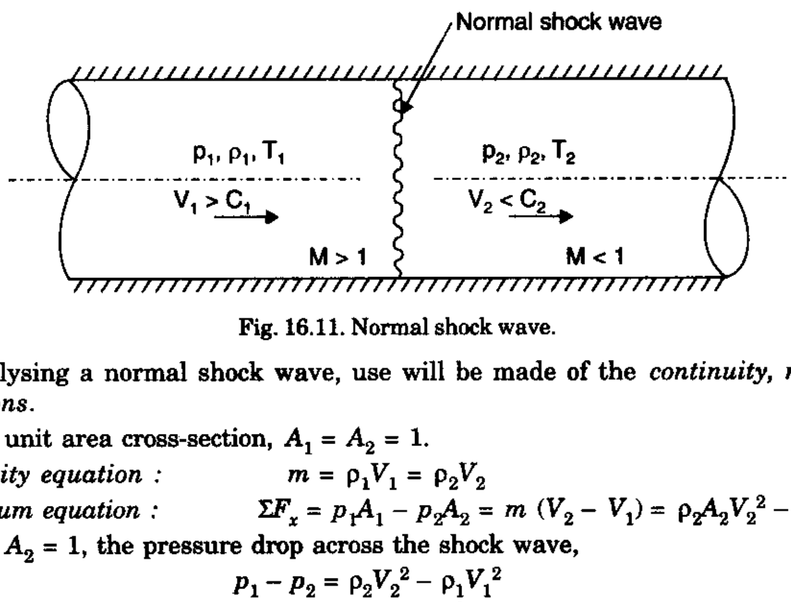


Fig. 16.11. Normal shock wave.

For analysing a normal shock wave, use will be made of the *continuity, momentum* and *energy equations*.

Assume unit area cross-section, $A_1 = A_2 = 1$.

Continuity equation : $m = \rho_1 V_1 = \rho_2 V_2$ ...(*i*)

Momentum equation : $\Sigma F_x = p_1 A_1 - p_2 A_2 = m\,(V_2 - V_1) = \rho_2 A_2 V_2^2 - \rho_1 A_1 V_1^2$

for $A_1 = A_2 = 1$, the pressure drop across the shock wave,

$$p_1 - p_2 = \rho_2 V_2^2 - \rho_1 V_1^2 \quad \text{...(ii)}$$

$$p_1 + \rho_1 V_1^2 = p_2 + \rho_2 V_2^2$$

Consider the flow across the shock wave as adiabatic.

Energy equation : $$\left(\frac{\gamma}{\gamma - 1}\right)\frac{p_1}{\rho_1} + \frac{V_1^2}{2} = \left(\frac{\gamma}{\gamma - 1}\right)\frac{p_2}{\rho_2} + \frac{V_2^2}{2} \quad \text{...[Eqn. (16.7)]}$$

($z_1 = z_2$, duct being in horizontal position)

or, $$\frac{\gamma}{\gamma - 1}\left(\frac{p_2}{\rho_2} - \frac{p_1}{\rho_1}\right) = \frac{V_1^2 - V_2^2}{2} \quad \text{...(iii)}$$

Combining continuity and momentum equations [refer eqns. (*i*) and (*ii*)], we get

$$p_1 + \frac{(\rho_1 V_1)^2}{\rho_1} = p_2 + \frac{(\rho_2 V_2)^2}{\rho_2} \quad \text{...(16.42)}$$

This equation is 'known as **Rankine Line Equation.**

Now combining continuity and energy equations [refer eqns. (*i*) and (*iii*)], we get

$$\frac{\gamma}{\gamma-1}\left(\frac{p_1}{\rho_1}\right)+\frac{(\rho_1 V_1)^2}{2\rho_1^2}=\frac{\gamma}{\gamma-1}\left(\frac{p_2}{\rho_2}\right)+\frac{(\rho_2 V_2)^2}{2\rho_2^2} \quad ...(16.43)$$

This equation is called **Fanno Line Equation.**

Further combining eqns. (*i*), (*ii*) and (*iii*) and solving for $\frac{p_2}{p_1}$, we get

$$\frac{p_2}{p_1}=\frac{\left(\frac{\gamma+1}{\gamma-1}\right)\frac{\rho_2}{\rho_1}-1}{\left(\frac{\gamma+1}{\gamma-1}\right)-\frac{\rho_2}{\rho_1}} \quad ...(16.44)$$

Solving for density ratio $\frac{\rho_2}{\rho_1}$, the same equations yield

$$\frac{\rho_2}{\rho_1}=\frac{V_1}{V_2}=\frac{1+\left(\frac{\gamma+1}{\gamma-1}\right)\frac{p_2}{p_1}}{\left(\frac{\gamma+1}{\gamma-1}\right)+\frac{p_2}{p_1}} \quad ...(16.45)$$

The eqns. (16.44) and (16.45) are called **Ranking-Hugoniot equations.**

One can also express $\frac{p_2}{p_1}$, $\frac{V_2}{V_1}$, $\frac{\rho_2}{\rho_1}$ and $\frac{T_2}{T_1}$ in terms of Mach number as follows :

$$\frac{p_2}{p_1}=\frac{2\gamma M_1^2-(\gamma-1)}{(\gamma+1)} \quad ...(16.46)$$

$$\frac{V_1}{V_2}=\frac{\rho_2}{\rho_1}=\frac{(\gamma+1)M_1^2}{(\gamma-1)M_1^2+2} \quad ...(16.47)$$

$$\frac{T_2}{T_1}=\frac{[(\gamma-1)M_1^2+2][2\gamma M_1^2-(\gamma-1)]}{(\gamma+1)^2 M_1^2} \quad ...(16.48)$$

By algebraic manipulation the following equation between M_1 and M_2 can be obtained.

$$M_2^2=\frac{(\gamma-1)M_1^2+2}{2\gamma M_1^2-(\gamma-1)} \quad ...(16.49)$$

Example 16.18. *For a normal shock wave in air Mach number is 2. If the atmospheric pressure and air density are 26.5 kN/m² and 0.413 kg/m³ respectively, determine the flow conditions before and after the shock wave. Take* $\gamma = 1.4$.

Sol. Let subscripts 1 and 2 represent the flow conditions before and after the shock wave.

Mach number, $M_1 = 2$

Atmospheric pressure, $p_1 = 26.5\ \text{kN/m}^2$

Air density, $\rho_1 = 0.413\ \text{kg/m}^3$

Mach number, M_2 :

$$M_2^2 = \frac{(\gamma - 1) M_1^2 + 2}{2\gamma M_1^2 - (\gamma - 1)} \quad \text{...[Eqn. (16.49)]}$$

$$= \frac{(1.4 - 1) \times 2^2 + 2}{2 \times 1.4 \times 2^2 - (1.4 - 1)} = \frac{3.6}{11.2 - 0.4} = 0.333$$

$\therefore$ $M_2 =$ **0.577 (Ans.)**

Pressure, p_2 :

$$\frac{p_2}{p_1} = \frac{2\gamma M_1^2 - (\gamma - 1)}{(\gamma + 1)} \quad \text{...[Eqn. (16.46)]}$$

$$= \frac{2 \times 1.4 \times 2^2 - (1.4 - 1)}{(1.4 + 1)} = \frac{11.2 - 0.4}{2.4} = 4.5$$

$\therefore$ $p_2 = 26.5 \times 4.5 =$ **119.25 kN/m² (Ans.)**

Density, ρ_2 :

$$\frac{\rho_2}{\rho_1} = \frac{(\gamma + 1) M_1^2}{(\gamma - 1) M_1^2 + 2} \quad \text{...[Eqn. (16.47)]}$$

$$= \frac{(1.4 + 1)2^2}{(1.4 - 1)2^2 + 2} = \frac{9.6}{1.6 + 2} = 2.667$$

$\therefore$ $\rho_2 = 0.413 \times 2.667 =$ **1.101 kg/m³ (Ans.)**

Temperature, T_1 :

Since $p_1 = \rho_1 RT_1$, $\therefore$ $T_1 = \frac{p_1}{\rho_1 R} = \frac{26.5 \times 10^3}{0.413 \times 287} = 223.6$ K or $-$ **49.4°C (Ans.)**

Temperature, T_2 :

$$\frac{T_2}{T_1} = \frac{[(\gamma - 1)M_1^2 + 2][2\gamma M_1^2 - (\gamma - 1)]}{(\gamma + 1)^2 M_1^2} = \frac{[(1.4 - 1)2^2 + 2][2 \times 1.4 \times 2^2 - (1.4 - 1)]}{(1.4 + 1)^2 2^2}$$

$$= \frac{(1.6 + 2)(11.2 - 0.4)}{23.04} = 1.6875$$

$\therefore$ $T_2 = 223.6 \times 1.6875 =$ **377.3 K** or **104.3°C (Ans.)**

Velocity, V_1 :

$$C_1 = \sqrt{\gamma RT_1} = \sqrt{1.4 \times 287 \times 223.6} = 299.7 \text{ m/s}$$

Since $\frac{V_1}{C_1} = M_1 = 2$ $\therefore$ $V_1 = 299.7 \times 2 =$ **599.4 m/s (Ans.)**

Velocity, V_2 :

$$C_2 = \sqrt{\gamma RT_2} = \sqrt{1.4 \times 287 \times 377.3} = 389.35 \text{ m/s}$$

Since $\frac{V_2}{C_2} = M_2 = 0.577$ $\therefore$ $V_2 = 389.35 \times 0.577 =$ **224.6 m/s (Ans.)**

16.11.2. Oblique Shock Wave

As shown in Fig. 16.12, when a supersonic flow undergoes a sudden turn through a small angle α (positive), an oblique wave is established at the corner. In comparison with normal shock waves, the oblique shock waves, being weaker, are preferred.

The shock waves should be avoided or made as weak as possible, since during a shock wave conversion of mechanical energy into heat energy takes place.

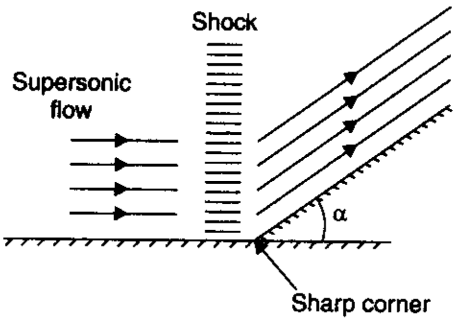


Fig. 16.12. Oblique shock wave.

16.11.3. Shock Strength

The *strength of shock* is defined as *the ratio of pressure rise across the shock to the upstream pressure.*

i.e. Strength of shock

$$= \frac{p_2 - p_1}{p_1} = \frac{p_2}{p_1} - 1$$

$$= \frac{2\gamma M_1^2 - (\gamma - 1)}{\gamma + 1} - 1 = \frac{2\gamma M_1^2 - (\gamma - 1) - (\gamma + 1)}{\gamma + 1}$$

$$= \frac{2\gamma M_1^2 - \gamma + 1 - \gamma - 1}{\gamma + 1} = \frac{2\gamma M_1^2 - 2\gamma}{\gamma + 1} = \frac{2\gamma}{\gamma + 1}(M_1^2 - 1)$$

Hence, strength of shock $= \frac{2\gamma}{\gamma + 1}(M_1^2 - 1)$ ...(16.50)

Example 16.19. *In a duct in which air is flowing, a normal shock wave occurs at a Mach number of 1.5. The static pressure and temperature upstream of the shock wave are 170 kN/m² and 23°C respectively. Determine :*

(*i*) *Pressure, temperature and Mach number downstream of the shock, and*

(*ii*) *Strength of shock.*

Take $\gamma = 1.4$.

Sol. Let subscripts 1 and 2 represent flow conditions upstream and downstream of the shock wave respectively.

Mach number, $M_1 = 1.5$

Upstream pressure, $p_1 = 170$ kN/m²

Upstream temperature, $T_1 = 23 + 273 = 296$ K

$\gamma = 1.4$

(*i*) **Pressure, temperature and Mach number downstream of the shock :**

$$\frac{p_2}{p_1} = \frac{2\gamma M_1^2 - (\gamma - 1)}{\gamma + 1} \quad \text{...[Eqn. (16.46)]}$$

$$= \frac{2 \times 1.4 \times 1.5^2 - (1.4 - 1)}{1.4 + 1} = \frac{6.3 - 0.4}{2.4} = 2.458$$

$\therefore$ $p_2 = 170 \times 2.458 =$ **417.86 kN/m². Ans.**

$$\frac{T_2}{T_1} = \frac{[(\gamma - 1)M_1^2 + 2][2\gamma M_1^2 - (\gamma - 1)]}{(\gamma + 1)^2 M_1^2} \quad \text{...[Eqn. (16.48)]}$$

$$= \frac{[(1.4-1)\times 1.5^2 + 2][2\times 1.4\times 1.5^2 - (1.4-1)]}{(1.4+1)^2 \times 1.5^2} = \frac{2.9\times 5.9}{12.96} = 1.32$$

$\therefore$ $T_2 = 296 \times 1.32 =$ **390.72 K or 117.72°C. Ans.**

$$M_2^2 = \frac{(\gamma-1)M_1^2 + 2}{2\gamma M_1^2 - (\gamma-1)} \qquad \text{...[Eqn. (16.49)]}$$

$$= \frac{(1.4-1)\times 1.5^2 + 2}{2\times 1.4\times 1.5^2 - (1.4-1)} = \frac{2.9}{5.9} = 0.49$$

$\therefore$ **M_2 = 0.7. Ans.**

(*ii*) **Strength of shock :**

Strength of shock $= \frac{p_2}{p_1} - 1 = 2.458 - 1 =$ **1.458. Ans.**

HIGHLIGHTS

1. A compressible flow is that flow in which the density of the fluid changes during flow.
2. The characteristic equation of state is given by :

$$\frac{p}{\rho} = RT$$

where p = absolute pressure, N/m²,
ρ = density of gas, kg/m³,
R = characteristic gas constant, J/kg K, and
R = absolute temperature (= t°C + 273).

3. The pressure and density of a gas are related as :

For isothermal process : $\frac{p}{\rho}$ = constant

For adiabatic process : $\frac{p}{\rho^\gamma}$ = constant.

4. The continuity equation for compressible flow is given as :

$$\rho AV = \text{constant}$$

$$\frac{d\rho}{\rho} + \frac{dA}{A} + \frac{dV}{V} = 0 \qquad \text{... in differential form.}$$

5. For compressible fluids Bernoulli's equation is given as :

$$\frac{p}{\rho g}\log_e p + \frac{V^2}{2g} + z = \text{constant} \qquad \text{... for isothermal process}$$

$$\left(\frac{\gamma}{\gamma-1}\right)\frac{p}{\rho g} + \frac{V^2}{2g} + z = \text{constant} \qquad \text{... for adiabatic process.}$$

6. Sonic velocity is given by :

$$C = \sqrt{\frac{dp}{d\rho}} = \sqrt{\frac{K}{\rho}} \qquad \text{... in terms of bulk modulus}$$

$$C = \sqrt{\frac{p}{\rho}} = \sqrt{RT} \qquad \text{... for isothermal process}$$

$$C = \sqrt{\frac{\gamma p}{\rho}} = \sqrt{\gamma RT} \qquad \text{... for adiabatic process.}$$

7. Mach number, $M = \frac{V}{C}$

(i) Subsonic flow : $M < 1$, $V < C$... disturbance always moves ahead of the projectile

(ii) Sonic flow : $M = 1$, $V = C$... disturbance moves along the projectile

(iii) Supersonic flow : $M > 1$, $V > C$... The projectile always moves ahead of the disturbance.

Mach angle is given by : $\sin\alpha = \frac{C}{V} = \frac{1}{M}$.

8. The pressure, temperature and density at a point where velocity is zero are called stagnation pressure (p_s), temperature, (T_s) and stagnation density ρ_s. Their values are given as :

$$p_s = p_o\left[1+\left(\frac{\gamma-1}{2}\right)M_0^2\right]^{\frac{\gamma}{\gamma-1}}$$

$$\rho_s = \rho_o\left[1+\left(\frac{\gamma-1}{2}\right)M_0^2\right]^{\frac{1}{\gamma-1}}$$

$$T_s = T_o\left[1+\left(\frac{\gamma-1}{2}\right)M_0^2\right]$$

where p_0, ρ_0 and T_o are the pressure, density and temperature at any point O in the flow.

9. Area-velocity relationship for compressible fluid is given as :

$$\frac{dA}{A} = \frac{dV}{V}(M^2 - 1)$$

(i) Subsonic flow ($M < 1$) : $\frac{dV}{V} > 0$; $\frac{dA}{A} < O$; $dp < 0$ (convergent nozzle)

$\frac{dV}{V} < 0$; $\frac{dA}{A} > 0$; $dp > 0$ (divergent diffuser)

(ii) Supersonic flow ($M > 1$) : $\frac{dV}{V} > 0$; $\frac{dA}{A} > 0$; $dp < 0$ (divergent nozzle)

$\frac{dV}{V} < 0$; $\frac{dA}{A} < 0$; $dp > 0$ (convergent diffuser)

(iii) Sonic flow ($M = 1$) : $\frac{dA}{A} = 0$ (straight flow passage since dA must be zero)

$dp = \frac{\text{zero}}{\text{zero}}$ *i.e.* indeterminate, but when evaluated,

the change of pressure $dp = 0$, since $dA = 0$ and the flow is frictionless.

10. Flow of compressible fluid through a convergent nozzle :

(i) Velocity through a nozzle or orifice fitted to a large tank :

$$V_2 = \sqrt{\left(\frac{2\gamma}{\gamma-1}\right)\frac{p_1}{\rho_1}\left[1-\left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}}\right]}$$

(*ii*) The mass rate of flow is given by :

$$m = A_2 \sqrt{\left(\frac{2\gamma}{\gamma - 1}\right) p_1 \rho_1 \left[\left(\frac{p_2}{p_1}\right)^{\frac{2}{\gamma}} - \left(\frac{p_2}{p_1}\right)^{\frac{\gamma+1}{\gamma}}\right]}$$

(*iii*) Value of $\left(\frac{p_2}{p_1}\right)$ for maximum value of mass rate of flow is given by :

$$\left(\frac{p_2}{p_1}\right) = \left(\frac{2}{\gamma + 1}\right)^{\frac{\gamma}{\gamma - 1}} = 0.528 \qquad \text{(when } \gamma = 1.4\text{)}$$

(*iv*) Value of V_2 for maximum rate of flow of liquid is given as,

$$V_2 = \sqrt{\left(\frac{2\gamma}{\gamma + 1}\right)\frac{p_1}{\rho_1}} \quad (= C_2)$$

(*v*) Maximum rate of flow of fluid through nozzle,

$$m_{max} = A_2 \sqrt{\left(\frac{2\gamma}{\gamma - 1}\right) p_1 \rho_1 \left[\left(\frac{2\gamma}{\gamma + 1}\right)^{\frac{2}{\gamma - 1}} - \left(\frac{2}{\gamma + 1}\right)^{\frac{\gamma+1}{\gamma - 1}}\right]}$$

For air, substituting $\gamma = 1.4$, we get

$$m_{max} = 0.685 A_2 \sqrt{p_1 \rho_1}$$

If the pressure ratio is less than 0.528, the mass rate of flow of the fluid is always corresponding to the pressure ratio of 0.528. But if the pressure ratio is more than 0.528, the mass rate of flow of fluid is corresponding to the given pressure ratio.

11. Whenever a supersonic flow (compressible) changes to subsonic flow, a shock wave (analogous to hydraulic jump in an open channel) is produced, resulting in a sudden rise in pressure, density, temperature and entropy.

$$p_1 + \frac{(\rho_1 V_1)^2}{\rho_1} = p_2 + \frac{(\rho_2 V_2)^2}{\rho_2} \qquad \text{... Ranking Line Equation}$$

$$\frac{\gamma}{\gamma - 1}\left(\frac{p_1}{\rho_1}\right) + \frac{(\rho_1 V_1)^2}{2\rho_1^2} = \frac{\gamma}{\gamma - 1}\left(\frac{p_2}{\rho_2}\right) + \frac{(\rho_2 V_2)^2}{2\rho_2^2} \qquad \text{... Fanno line Equation}$$

$$\left.\begin{aligned} \frac{p_2}{p_1} &= \frac{\left(\frac{\gamma+1}{\gamma-1}\right)\frac{\rho_2}{\rho_1} - 1}{\left(\frac{\gamma+1}{\gamma-1}\right) - \frac{\rho_2}{\rho_1}} \\ \frac{\rho_2}{\rho_1} = \frac{V_1}{V_2} &= \frac{1 + \left(\frac{\gamma+1}{\gamma-1}\right)\frac{p_2}{p_1}}{\left(\frac{\gamma+1}{\gamma-1}\right) + \frac{p_2}{p_1}} \end{aligned}\right\} \qquad \text{...Rankinge-Hugoniot Equations}$$

One can also express $\frac{p_2}{p_1}, \frac{V_2}{V_1}, \frac{\rho_2}{\rho_1}$ and $\frac{T_2}{T_1}$ in terms of Mach number as follows :

$$\frac{p_2}{p_1} = \frac{2\gamma M_1^2 - (\gamma - 1)}{\gamma + 1} \qquad \text{...(i)}$$

$$\frac{V_1}{V_2} = \frac{\rho_2}{\rho_1} = \frac{(\gamma+1)M_1^2}{(\gamma-1)M_1^2+2} \qquad ...(ii)$$

$$\frac{T_2}{T_1} = \frac{[(\gamma-1)M_1^2+2][2\gamma M_1^2-(\gamma-1)]}{(\gamma+1)^2 M_1^2} \qquad ...(iii)$$

Also $$M_2^2 = \frac{(\gamma-1)M_1^2+2}{2\gamma M_1^2-(\gamma-1)}.$$

OBJECTIVE TYPE QUESTIONS

Choose the Correct Answer :

1. All real fluids are
(*a*) incompressible (*b*) compressible to some extent
(*c*) compressible to any extent (*d*) none of the above.
2. A change in the state of a system at constant volume is called
(*a*) isobaric process (*b*) isochoric process
(*c*) isothermal process (*d*) adiabatic process.
3. A process during which no heat is transferred to or from the gas is called an
(*a*) isochoric process (*b*) isobaric process
(*c*) adiabatic process (*d*) isothermal process.
4. An adiabatic process is one which follows the relation
(*a*) $\frac{p}{\rho}$ = constant (*b*) $\frac{p}{\rho^\gamma}$ = constant
(*c*) $\frac{p}{\rho^n}$ = constant ($n \neq \gamma$) (*d*) v = constant.
5. An isentropic flow is one which is
(*a*) isothermal (*b*) adiabatic
(*c*) adiabatic and irreversible (*d*) adiabatic and reversible.
6. Indica upto what Mach number can a fluid flow be considered incompressible ?
(*a*) 0.1 (*b*) 0.3
(*c*) 0.8 (*d*) 1.0.
7. Which of the following is the basic equation of compressible fluid flow ?
(*a*) Continuity equation (*b*) Momentum equation
(*c*) Energy equation (*d*) Equation of state
(*e*) All of the above.
8. The velocity of disturbance in case of fluids is ...... the velocity of the disturbance in solids.
(*a*) less than (*b*) equal to
(*c*) more than (*d*) none of the above.
9. Sonic velocity (C) for adiabatic process is given as
(*a*) $C = \sqrt{\gamma R T^3}$ (*b*) $C = \sqrt{\gamma RT}$
(*c*) $C = \sqrt{\gamma^2 RT}$ (*d*) $C = \gamma RT$.
where γ = ratio of specific heats, R = gas constant, T = temperature.
10. The flow is said to be subsonic when Mach number is
(*a*) equal to unity (*b*) less than unity
(*c*) greater than unity (*d*) none of the above.

11. The region outside the Mach cone is called
(a) zone of action
(b) zone of silence
(c) control volume
(d) none of the above.

12. A stagnation point is the point on the immersed body where the magnitude of velocity is
(a) small
(b) large
(c) zero
(d) none of the above.

13. A convergent-divergent nozzle is used when the discharge pressure is
(a) less than the critical pressure
(b) equal to the critical pressure
(c) more than the critical pressure
(d) none of the above.

14. At critical pressure ratio, the velocity at the throat of a nozzle is
(a) equal to the sonic speed
(b) less than the sonic speed
(c) more than the sonic speed
(d) none of the above.

15. Laval nozzle is a
(a) convergent nozzle
(b) divergent nozzle
(c) convergent-divergent nozzle
(d) any of the above.

16. A shock wave is produced when
(a) a subsonic flow changes to sonic flow
(b) a sonic flow changes to supersonic flow
(c) a supersonic flow changes to subsonic flow
(d) none of the above.

17. The sonic velocity in a fluid medium is directly proportional to
(a) Mach number
(b) pressure
(c) square root of temperature
(d) none of the above.

18. The stagnation pressure (p_s) and temperature (T_s) are
(a) less than their ambient counterparts
(b) more than their ambient counterparts
(c) the same as in ambient flow
(d) none of the above.

19. Across a normal shock
(a) the entropy remains constant
(b) the pressure and temperature rise
(c) the velocity and pressure decrease
(d) the density and temperature decrease.

20. A normal shock wave
(a) is reversible
(b) is irreversible
(c) is isentropic
(d) occurs when approaching flow is supersonic.

21. The sonic speed in an ideal gas varies
(a) inversely as bulk modulus
(b) directly as the absolute pressure
(c) inversely as the absolute temperature
(d) none of the above.

22. In a supersonic flow, a diffuser is a conduit having
(a) gradually decreasing area
(b) converging-diverging passage
(c) constant area throughout its length
(d) none of the above.

23. Choking of a nozzle fitted to a pressure tank containing gas implies
(a) sonic velocity at the throat
(b) increase of the mass flow rate
(c) obstruction of flow
(d) all of the above.

24. A shock wave which occurs in a supersonic flow represents a region in which
(a) a zone of silence exists
(b) there is no change in pressure, temperature and density
(c) there is sudden change in pressure, temperature and density
(d) velocity is zero.

25. Which of the following statements regarding a normal shock is *correct* ?
(a) It occurs when an abrupt change takes place from supersonic into subsonic flow condition
(b) It causes a disruption and reversal of flow pattern
(c) It may occur in sonic or supersonic flow
(d) None of the above.

26. For compressible fluid flow the area-velocity relationship is

(a) $\frac{dA}{A} = \frac{dV}{V}(1 - M^2)$ (b) $\frac{dA}{A} = \frac{dV}{V}(C^2 - 1)$

(c) $\frac{dA}{A} = \frac{dV}{V}(M^2 - 1)$ (d) $\frac{dA}{A} = \frac{dV}{V}(1 - V^2)$.

27. The sonic velocity is largest in which of the following ?

(a) Water (b) Steel

(c) Kerosene (d) Air.

28. Which of the following expressions does not represent the speed of sound in a medium ?

(a) $\sqrt{\frac{K}{\rho}}$ (b) $\sqrt{\gamma RT}$

(c) $\sqrt{K\frac{p}{\rho}}$ (d) $\sqrt{\frac{dp}{d\rho}}$.

29. The differential equation for energy in isentropic flow is of the form

(a) $\frac{dV}{V} + \frac{dp}{\rho} + \frac{dA}{A} = 0$ (b) $VdV + \frac{dp}{\rho} = 0$

(c) $2VdV + \frac{dp}{\rho} = 0$ (d) $dp + d(\rho V^2) = 0$.

30. Which of the following statements is *incorrect* ?

(a) A shock wave occurs in divergent section of a nozzle when the compressible flow changes abruptly from supersonic to subsonic state

(b) A plane moving at supersonic state is not heard by the stationary observer on the ground until it passes him because zone of disturbance in Mach cone trails behind the plane

(c) A divergent section is added to a convergent nozzle to obtain supersonic velocity at the throat

(d) none of the above.

ANSWERS

1. (b)	2. (b)	3. (c)	4. (b)	5. (d)	6. (b)	7. (e)
8. (a)	9. (b)	10. (b)	11. (b)	12. (c)	13. (a)	14. (a)
15. (c)	16. (c)	17. (c)	18. (b)	19. (b)	20. (d)	21. (d)
22. (a)	23. (d)	24. (c)	25. (a)	26. (c)	27. (b)	28. (c)
29. (b)	30. (c).					

THEORETICAL QUESTIONS

1. Differentiate between compressible and incompressible flows.
2. Give the examples when liquid is treated as a compressible fluid.
3. When is the compressibility of fluid important ?
4. What is the difference between isentropic and adiabatic flows ?
5. What is the relation between pressure and density of a compressible fluid for (a) isothermal process (b) adiabatic process ?
6. Obtain an expression in differential form for continuity equation for one-dimensional compressible flow.
7. Derive an expression for Bernoulli's equation when the process is adiabatic.
8. How are the disturbances in compressible fluid propagated ?
9. What is sonic velocity ? On what factors does it depend ?
10. What is Mach number ? Why is this parameter so important for the study of flow of compressible fluids ?

11. Prove that velocity of sound wave in a compressible fluid is given by : $C = \sqrt{k/\rho}$, where k and ρ are the bulk modulus and density of fluid respectively.
12. Define the following terms :
(*i*) Subsonic flow (*ii*) Sonic flow (*iii*) Supersonic flow, (*iv*) Mach cone.
13. What is silence zone during the disturbance which propagates when an object moves in still air ?
14. What is stagnation point of an object immersed in fluid ?
15. What is stagnation pressure ?
16. What are static and stagnation temperatures ?
17. Derive an expression for mass flow rate of compressible fluid through an orifice or nozzle fitted to a large tank. What is the condition for maximum rate of flow ?
18. What is the critical pressure ratio for a compressible flow through a nozzle ? On what factors does it depend ?
19. Describe compressible flow through a convergent-divergent nozzle. How and where does the shock wave occur in the nozzle ?
20. What do you mean by compressibility correction factor ?
21. How is a shock wave produced in a compressible fluid ? What do you mean by the term "Shock strength" ?

UNSOLVED EXAMPLES

1. A 100 mm diameter pipe reduces to 50 mm diameter through a sudden contraction. When it carries air at 20.16°C under isothermal conditions, the absolute pressures observed in the two pipes just before and after the contraction are 400 kN/m² and 320 kN/m² respectively. Determine the densities and velocities at the two sections. Take R = 290 J/kg K. [**Ans.** 4.7 kg/m³ ; 3.76 kg/m³ ; 39.7 m/s ; 198.5 m/s]
2. A gas with a velocity of 300 m/s is flowing through a horizontal pipe at a section where pressure is 60 kN/m² (*abs.*) and temperature 40°C. The pipe changes in diameter and at this section the pressure is 90 kN/m². If the flow of gas is adiabatic find the velocity of gas at this section.
Take : R = 287 J/kg K and γ = 1.4. [**Ans.** 113 m/s]
3. An aeroplane is flying at 21.5 m/s at a low altitude where the velocity of sound is 325 m/s. At a certain point just outside the boundary layer of the wings, the velocity of air relative to the plane is 305 m/s. If the flow is frictionless adiabatic determine the pressure drop on the wing surface near this position.
Assume γ = 1.4, pressure of ambient air = 102 kN/m². [**Ans.** 28.46 kN/m²]
4. A jet propelled aircraft is flying at 1100 km/h. at sea level. Calculate the Mach number at a point on the aircraft where air temperature is 20°C.
Take : R = 287 J/kg K and γ = 1.4. [**Ans.** 0.89]
5. An aeroplane is flying at an height of 20 km where the temperature is – 40°C. The speed of the plane is corresponding to M = 1.8. Find the speed of the plane.
Take : R = 287 J/kg K, γ = 1.4. [**Ans.** 1982.6 km/h]
6. Find the velocity of bullet fired in standard air if its Mach angle is 30°. [**Ans.** 680.4 m/s]
7. Air, thermodynamic state of which is given by pressure p = 230 kN/m² and temperature = 300 K is moving at a velocity V = 250 m/s. Calculate the stagnation pressure if (*i*) compressibility is neglected and (*ii*) compressibility is accounted for.
Take γ = 1.4 and R = 287 J/kg K. [**Ans.** 313 kN/m², 323 kN/m²]
8. A large vessel, fitted with a nozzle, contains air at a pressure of 2943 kN/m² (*abs.*) and at a temperature of 20°C. If the pressure at the outlet of the nozzle is 2060 kN/m² (*abs.*) find the velocity of air flowing at the outlet of the nozzle.
Take : R = 287 J/kg K and γ = 1.4 [**Ans.** 239.2 m/s]
9. Nitrogen gas (γ = 1.4) is released through a 10 mm orifice on the side of a large tank in which the gas is at a pressure of 10 bar and temperature 20°C. Determine the mass flow rate if (*i*) the gas escapes to atmosphere (1 bar) ; (*ii*) the gas is released to another tank at (*a*) 5 bar, (*b*) 6 bar.
[**Ans.** (*i*) 0.183 kg/s ; (*ii*) 0.183 kg/s ; 0.167 kg/s]

10. Air is released from one tank to another through a convergent-divergent nozzle at the rate of 12 N/s. The supply tank is at a pressure of 400 kN/m^2 and temperature 110°C, and the pressure in the receiving tank is 100 kN/m^2. Determine : (*i*) The pressure, temperature, and Mach number in the constriction, (*ii*) The required diameter of constriction, (*iii*) The diameter of the nozzle at the exit for full expansion, and the Mach number. [**Ans.** (*i*) 210 kN/m^2 ; 319 K, (*ii*) 43.5 mm ; (*iii*) 48 mm ; 1.56]

11. Oxygen flows in a conduit at an absolute pressure of 170 kN/m^2. If the absolute pressure and temperature at the nose of small object in the stream are 200 kN/m^2 and 70.16°C respectively, determine the velocity in the conduit. Take $\gamma = 1.4$ and $R = 281.43$ J/kg K. [**Ans.** 175.3 m/s]

12. Air at a velocity of 1400 km/h has a pressure of 10 kN/m^2 vacuum and temperature of 50.16°C. Calculate local Mach number and stagnation pressure, density and temperature. Take $\gamma = 1.4$, $R = 281.43$ J/kg K and barometric pressure = 101.325 kN/m^2. [**Ans.** 1.089 ; 192.358 kN/m^2 ; 1.708 kg/m^3 ; 399.8 K]

13. A normal shock wave occurs in a diverging section when air is flowing at a velocity of 420 m/s, pressure 100 kN/m^2, and temperature 10°C. Determine : (*i*) The Mach number before and after the shock, (*ii*) The pressure rise, and (*iii*) The velocity and temperature after the shock.

[**Ans.** (*i*) 1.25 ; 0.91 ; (*ii*) 66 kN/m^2, (*iii*) 292 m/s ; 54°C]

14. A normal shock wave occurs in air flowing at a Mach number of 1.5. The static pressure and temperature of the air upstream of the shock wave are 100 kN/m^2 and 300 K. Determine the Mach number, pressure and temperature downstream of the shockwave. Also estimate the shock strength.

[**Ans.** 0.7 ; 246 kN/m^2 ; 396.17 K ; 1.46]

COMPETITIVE EXAMINATIONS QUESTIONS

(Including ESE and CSE Questions, from 1996 onwards)

Match List I with List II or Choose the Correct Answer :

1. Match List I with List II and select the correct answer using the codes given below the lists :

List I	List II
A. Work done in a polytropic process	1. $\int v\,dp$
B. Work done in a steady flow process	2. Zero
C. Heat transfer in a reversible adiabatic process	3. $\dfrac{p_1V_1 - p_2V_2}{\gamma - 1}$
D. Work done in an isentropic process	4. $\dfrac{p_1V_1 - p_2V_2}{n - 1}$

Codes :

	A	B	C	D
(*a*)	4	1	3	2
(*b*)	1	4	2	3
(*c*)	4	1	2	3
(*d*)	1	2	3	4.

2. Match the curves in Fig. 1 with the curves in Fig. 2 and select the correct answer using the codes given below the diagrams :

Diagram I (Process on *p-V* plane)

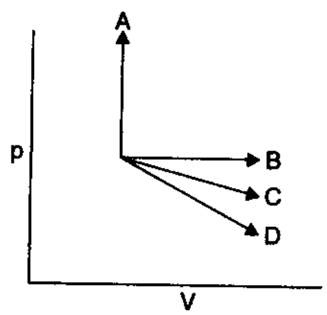


Fig. 1

Diagram II (Process on *T-s* plane)

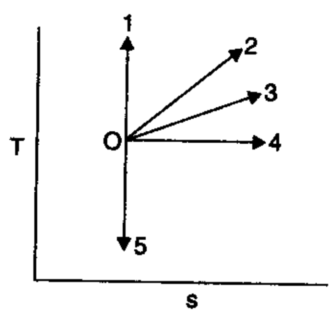


Fig. 2

Codes :

	A	B	C	D
(*a*)	3	2	4	5
(*b*)	2	3	4	5
(*c*)	2	3	4	1
(*d*)	1	4	2	3.

3. The heat transfer Q, the work done W and the change in internal energy ΔU are all zero in the case of
   (*a*) a rigid vessel containing steam at 150°C left in the atmosphere which is at 25°C.
   (*b*) 1 kg of gas contained in an insulated cylinder expanding as the piston move slowly outwards.
   (*c*) a rigid vessel containing ammonia gas connected through a valve to an evacuated rigid vessel, the vessel, the valve and the connecting pipes being well insulated and the valve being opened and after a time, conditions through the two vessel becoming uniform.
   (*d*) 1 kg of air flowing adiabatically from the atmosphere into a previously evacuated bottle.
4. Zeroth Law of thermodynamics states that
   (*a*) two thermodynamic system are always in thermal equilibrium with each other.
   (*b*) if two systems are in thermal equilibrium, then the third system will also be in thermal equilibrium.
   (*c*) two systems not in thermal equilibrium with a third system are also not in thermal equilibrium with each other.
   (*d*) when two systems are in thermal equilibrium with a third system, they are in thermal equilibrium with each other.
5. Which one of the following statements applicable to a perfect gas will also be true for an irreversible process ? (Symbols have the usual meanings)
   (*a*) $dQ = du + pdV$ (*b*) $dQ = Tds$
   (*c*) $Tds = du + pdV$ (*d*) None of the above.
6. The throttling process undergone by a gas across an orifice is shown by its states in Fig. 3.

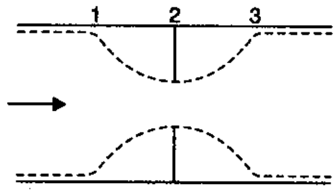


Fig. 3

It can be represented on the *T-s* diagram as

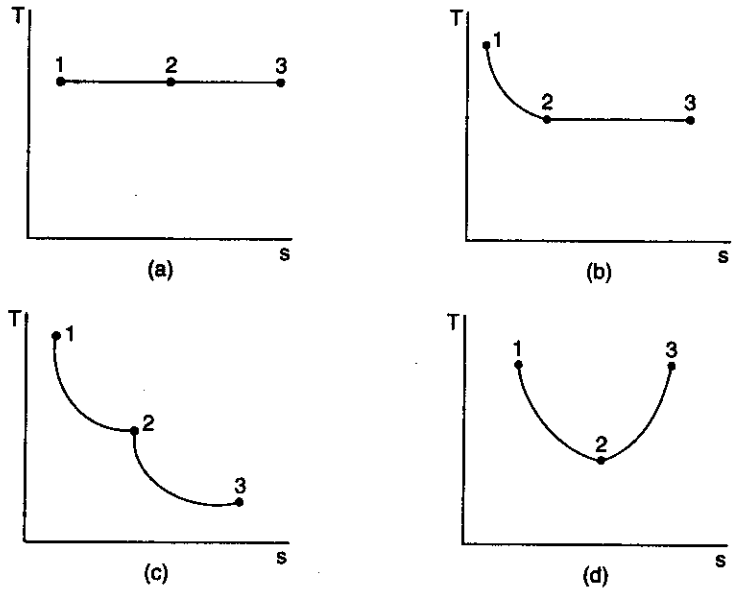


Fig. 4

7. Which one of the following temperature entropy diagrams of steam shows the reversible and irreversible processes correctly ?

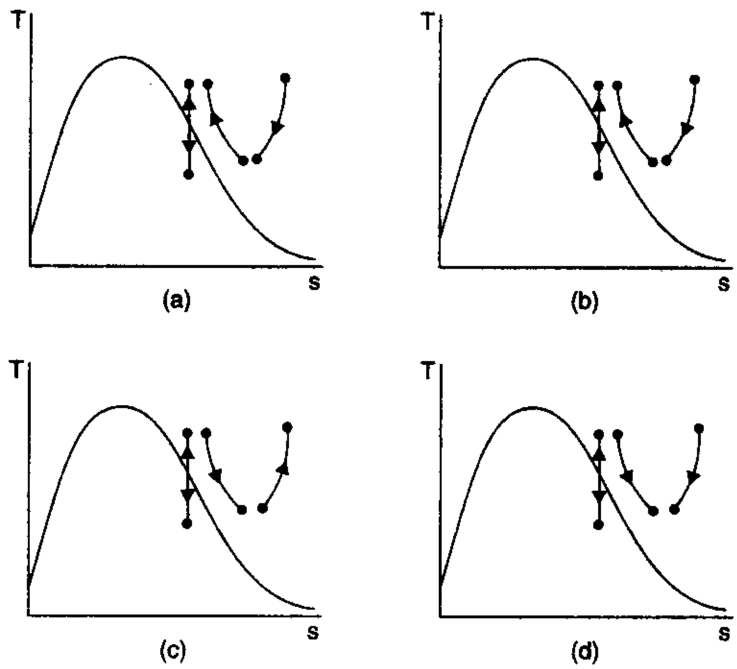


Fig. 5

8. Consider the following statements :

1. Availability is generally conserved.
2. Availability can either be negative or positive.
3. Availability is the maximum theoretical work obtainable.
4. Availability can be destroyed in irreversibilities.

Of these statements

(*a*) 3 and 4 are correct (*b*) 1 and 3 are correct

(*c*) 1 and 4 are correct (*d*) 2 and 4 are correct.

9. For a given volume of dry saturated steam, Clapeyron's equation is given by

(*a*) $V_g - V_t = \frac{dT_s}{dp} \times \frac{T}{h_g - h_f}$ (*b*) $V_g - V_f = \frac{dT_s}{dp} \times \frac{h_g - h_f}{T_s}$

(*c*) $V_g - V_f = \frac{dp}{dT_s} \times \frac{h_g - h_f}{T_s}$ (*d*) $V_g - V_f = \frac{dp}{dT_s} \times \frac{T_s}{h_g - h_f}$.

10. The Joule-Thomson coefficient is the

(*a*) $\left(\frac{\partial T}{\partial p}\right)_h$ of pressure-temperature curve of real gases

(*b*) $\left(\frac{\partial T}{\partial s}\right)_v$ of temperature entropy curve of real gases

(c) $\left(\frac{\partial h}{\partial s}\right)_T$ of enthalpy entropy curve of real gases

(d) $\left(\frac{\partial v}{\partial T}\right)_p$ of pressure volume curve of real gases.

11. Which one of the following *p-T* diagrams illustrates the Otto cycle of an ideal gas ?

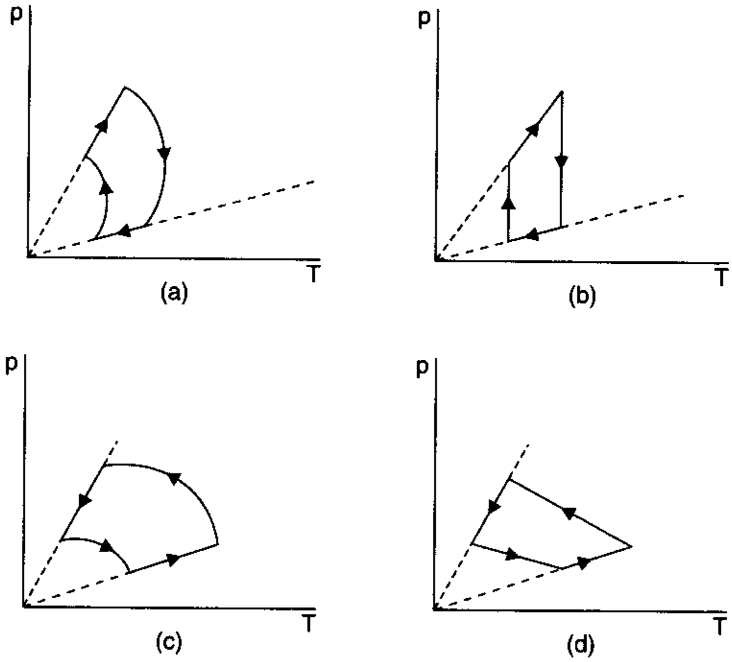


Fig. 6

12. When a system is taken from state *A* to state *B* along the path *A-C-B*, 180 kJ of heat flows into the system and it does 130 kJ of work (see Fig. 7 given).

How much heat will flow into the system along the path *A-D-B* if the work done by it along the path is 40 kJ ?

(a) 40 kJ (b) 60 kJ
(c) 90 kJ (d) 135 kJ.

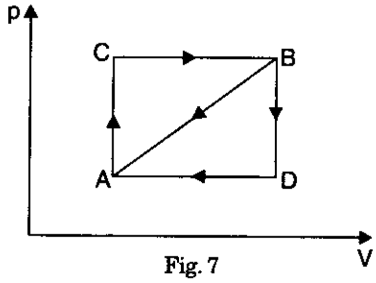


Fig. 7

13. A gas expands from pressure p_1 to pressure p_2 $\left(p_2 = \frac{p_1}{10}\right)$. If the process of expansion is isothermal, the volume at the end of expansion is 0.55 m^3. If the process of expansion is adiabatic, the volume at the end of expansion will be closer to

(a) 0.45 m^3 (b) 0.55 m^3
(c) 0.65 m^3 (d) 0.75 m^3.

14. A standard vapour is compressed to half its volume without changing is temperature. The result is that :

(*a*) All the vapour condenses to liquid

(*b*) Some of the liquid evaporates and the pressure does not change

(*c*) The pressure is double its initial value

(*d*) Some of the vapour condenses and the pressure does not change.

15. A system of 100 kg mass undergoes a process in which its specific entropy increases from 0.3 kJ/kg-K to 0.4 kJ/kg-K. At the same time, the entropy of the surroundings decreases from 80 kJ/K to 75 kJ/K. The process is

(*a*) Reversible and isothermal (*b*) Irreversible

(*c*) Reversible (*d*) Impossible.

16. The thermodynamic parameters are :

I. Temperature II. Specific volume

III. Pressure IV. Enthaply

V. Entropy

The Clapeyron equation of state provides relationship between

(*a*) I and II (*b*) II, III and V

(*c*) III, IV and V (*d*) I, II, III and IV.

17. The work done in compressing a gas isothermally is given by :

(*a*) $\frac{\gamma}{\gamma - 1} \cdot p_1V_1\left[\left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}} - 1\right]$ (*b*) $mRT_1 \log_e \frac{p_2}{p_1}$ N.m

(*c*) $mc_p (T_2 - T_1)$ kJ (*d*) $mRT_1 \left(1 - \frac{T_2}{T_1}\right)$ kJ.

18. An ideal air standard cycle is shown in the given temperature entropy diagram (Fig. 8).

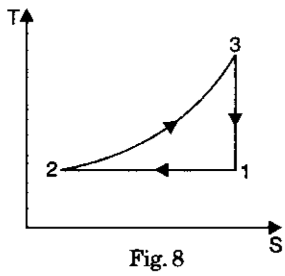
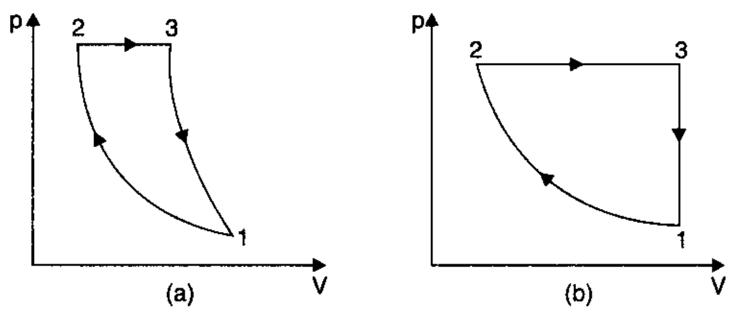


Fig. 8

The same cycle, when represented on the pressure-volume co-ordinates, takes the form.



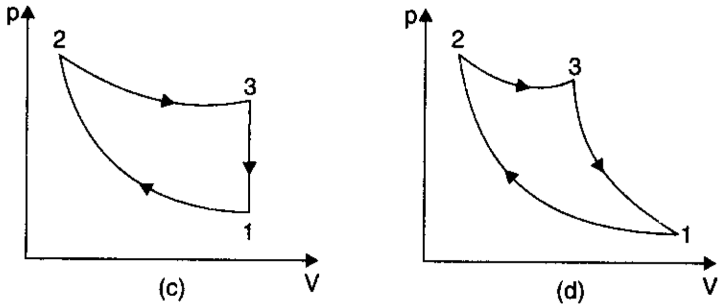


Fig. 9

19. In a Rankine cycle, with the maximum steam temperature being fixed from metallurgical considerations, as the boiler pressure increases

(*a*) the condenser load will increase

(*b*) the quality of turbine exhaust will decrease

(*c*) the quality of turbine exhaust will increase

(*d*) the quality of turbine exhaust will remain unchanged.

20. Match List I (details of the processes of the cycle) with List II (name of the cycle) and select the correct answer using the code given below the Lists :

List I	List II
A. Two isothermals and two adiabatics	1. Otto
B. Two isothermals and two constant volumes	2. Joule
C. Two adiabatics and two constant pressures	3. Carnot
D. Two adiabatics and two constant pressures	4. Stirling

Codes :

	A	*B*	*C*	*D*		*A*	*B*	*C*	*D*
(*a*)	4	3	1	2	(*b*)	4	3	2	1
(*c*)	3	4	1	2	(*d*)	3	4	2	1.

21. Two blocks which are at different states are brought into contact with each other and allowed to reach a final state of thermal equilibrium. The final temperature attained is specified by the

(*a*) Zeroth law of thermodynamics (*b*) First law of thermodynamics

(*c*) Second law of thermodynamics (*d*) Third law of thermodynamics.

22. A control mass undergoes a process from state 1 to state 2 as shown in Fig. 10. During this process, the heat transfer to the system is 200 kJ. If the control mass returned adiabatically from state 2 to state 1 by another process, then the work interaction during the return process (in kN.m) would be

(*a*) – 400 (*b*) – 200

(*c*) 200 (*d*) 400.

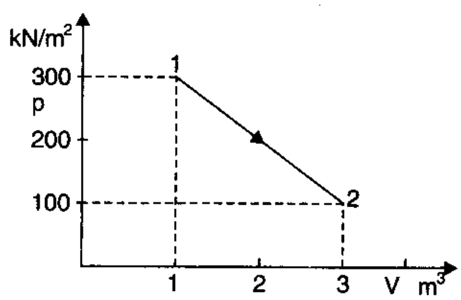


Fig. 10

23. Four processes of a thermodynamic cycle are shown above in Fig. 11 on the *T-s* plane in the sequence 1—2—3—4. The corresponding correct sequence of these processes in the *p-V* plane as shown in Fig. 12 will be

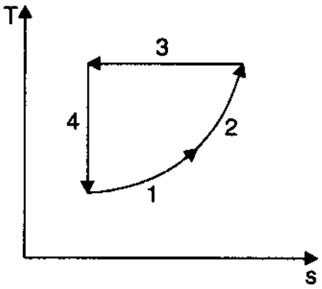


Fig. 11

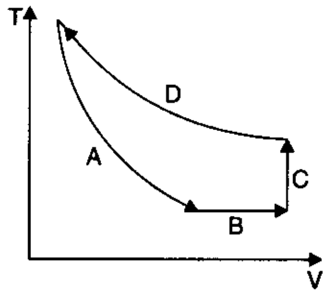


Fig. 12

(*a*) (*C—D—A—B*)
(*b*) (*D—A—B—C*)
(*c*) (*A—B—C—D*)
(*d*) (*B—C—D—A*).

24. The Fig. 13 shows an isometric cooling process 1-2 of a pure substance. The ordinate and abscissa are respectively
(*a*) pressure and volume
(*b*) enthalpy and entropy
(*c*) temperature and entropy
(*d*) pressure and enthalpy.

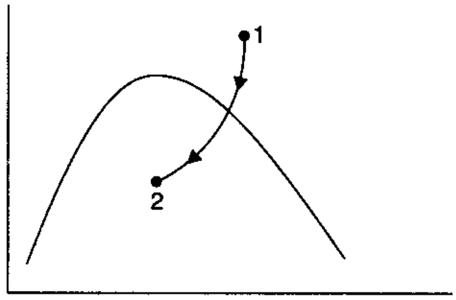


Fig. 13

25. For a thermodynamic cycle to be irreversible, it is necessary that

(*a*) $\int \frac{\delta Q}{T} = 0$
(*b*) $\int \frac{\delta Q}{T} < 0$
(*c*) $\int \frac{\delta Q}{T} > 0$
(*d*) $\int \frac{\delta Q}{T} \geq 0$.

26. Neglecting changes in kinetic energy and potential energy, for unit mass the availability in a non-flow process becomes $a = \phi - \phi_0$ where ϕ is the availability function of the
(*a*) open system
(*b*) closed system
(*c*) isolated system
(*d*) steady flow process.

27. It can be shown that for a simple compressible substance, the relationship

$$c_p - c_v = -T\left(\frac{\partial V}{\partial T}\right)_P^2 \left(\frac{\partial p}{\partial v}\right)_T \quad \text{exists}$$

where c_p and c_v are specific heats at constant pressure and constant volume respectively, T is temperature, *V* is volume and *p* is pressure.

Which one of the following statements is NOT true ?

(*a*) c_p is always greater than c_v
(*b*) The right side of the equation reduces to R for an ideal gas

(*c*) Since $\left(\frac{\partial p}{\partial V}\right)_T$ can be either positive or negative, and $\left(\frac{\partial V}{\partial T}\right)_p$ must be positive, T must have a sign which is opposite to that of $\left(\frac{\partial p}{\partial V}\right)_T$

(*d*) c_p is very nearly equal to c_v for liquid water.

28. Consider the following statements : In an irreversible process

1. entropy always increases.
2. the sum of the entropy of all the bodies taking part in a process always increases.
3. once created, entropy cannot be destroyed.

Of these statements :

(*a*) 1 and 2 are correct (*b*) 1 and 3 are correct

(*c*) 2 and 3 are correct (*d*) 1, 2 and 3 are correct.

29. An ideal cycle is shown in the Fig. 14. Its thermal efficiency is given by

(*a*) $1-\dfrac{\left(\dfrac{v_3}{v_1}-1\right)}{\left(\dfrac{p_2}{p_1}-1\right)}$ (*b*) $1-\dfrac{1}{\gamma}\dfrac{\left(\dfrac{v_3}{v_1}-1\right)}{\left(\dfrac{p_2}{p_1}-1\right)}$

(*c*) $1-\gamma\dfrac{(v_3-v_1)}{(p_2-p_1)}\dfrac{p_1}{v_1}$ (*d*) $1-\dfrac{1}{\gamma}\dfrac{(p_3-p_1)}{(v_3-v_1)}\dfrac{v_1}{p_1}$.

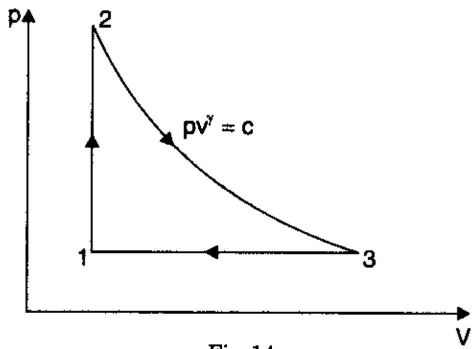


Fig. 14

30. Consider the following statements regarding Otto cycle :

1. It is not a reversible cycle.
2. Its efficiency can be improved by using a working fluid of higher value of ratio of specific heats.
3. The practical way of increasing its efficiency is to increase the compression ratio.
4. Carburetted gasoline engines working on Otto cycle can work with compression ratios more than 12.

Of these statements :

(*a*) 1, 3 and 4 are correct (*b*) 1, 2 and 3 are correct

(*c*) 1, 2 and 4 are correct (*d*) 2, 3 and 4 are correct.

31. Consider the following statements : The difference between higher and lower heating values of the fuels is due to

1. heat carried by steam from the moisture content of fuel.
2. sensible heat carried away by the flue gases.
3. heat carried away by steam from the combustion of hydrogen in the fuel.
4. heat lost by radiation.

Of these statements :

(*a*) 2, 3 and 4 are correct (*b*) 1 and 2 are correct

(*c*) 3 alone is correct (*d*) 1, 2, 3 and 4 are correct.

32. Match List I (Gadgets undergoing a thermodynamic process) with List II (Property of the system that remains constant) and select the correct answer using the codes given below the Lists :

List I	**List II**
A. Bomb calorimeter	1. Pressure
B. Exhaust gas calorimeter	2. Enthalpy
C. Junker gas calorimeter	3. Volume
D. Throttling calorimeter	4. Specific heats

Codes :

	A	*B*	*C*	*D*		*A*	*B*	*C*	*D*
(*a*)	3	4	1	2	(*b*)	2	4	1	3
(*c*)	3	1	4	2	(*d*)	4	3	2	1

33. Consider the following statements :

The maximum temperature produced by the combustion of an unit mass of fuel depends upon

1. LCV 2. ash content 3. mass of air supplied 4. pressure in the furnace.

Of these statements :

(*a*) 1 alone is correct (*b*) 1 and 3 are correct

(*c*) 2 and 4 are correct (*d*) 3 and 4 are correct.

34. The graph shown in the Fig. 15 represents the emission of a pollutant from an SI engine for different fuel/air ratios. The pollutant in question is

(*a*) CO

(*b*) CO_2

(*c*) hydrocarbon

(*d*) NO_x.

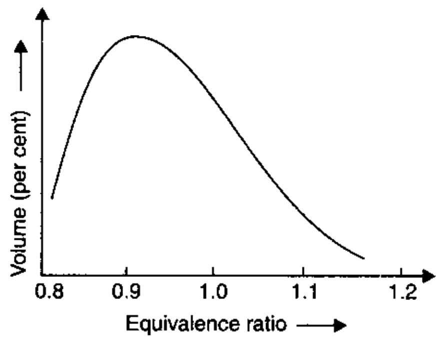


Fig. 15

35. Which of the following are the assumptions involved in the auto-ignition theory put forth for the onset of knock in SI engines ?

1. Flame velocity is normal before the onset of auto-ignition.
2. A number of end-gas elements autoignite simultaneously.
3. Preflame reactions are responsible for preparing the end-gas to ignite.

Select the correct answer using the codes given below :

(*a*) 1 and 2 (*b*) 1 and 3

(*c*) 2 and 3 (*d*) 1, 2 and 3.

36. In a vapour compression refrigeration system, a throttle valve is used in place of an expander because

(*a*) it considerably reduces the system weight

(*b*) it improves the COP, as the condenser is small

(*c*) the positive work in isentropic expansion of liquid is very small

(*d*) it leads to significant cost reduction.

37. A cube at high temperature is immersed in a constant temperature bath. It loses heat from its top, bottom and side surfaces with heat transfer coefficient of h_1, h_2 and h_3, respectively. The average heat transfer coefficient for the cube is

(*a*) $h_1 + h_2 + h_3$ (*b*) $(h_1 h_2 h_3)^{1/3}$

(*c*) $\frac{1}{h_1} + \frac{1}{h_2} + \frac{1}{h_3}$ (*d*) None of the above.

38. Match items in List I with those in Lists II and III and select the correct answer using the codes given the lists :

List I	List II	List III
A. Reversed Carnot engine	1. Condenser	6. Generator
B. Sub-cooling	2. Evaporator	7. Increase in refrigeration effect
C. Superheating	3. Vortex refrigerator	8. Highest COP
D. Constant enthalpy	4. Throttling	9. Adiabatic
	5. Heat pump	10. Dry compression

Codes :

	A	*B*	*C*	*D*
(*a*)	3, 10	1, 7	2, 9	4, 6
(*b*)	5, 8	1, 7	2, 10	4, 9
(*c*)	4, 10	3, 8	3, 10	1, 6
(*d*)	2, 7	5, 8	4, 6	1, 9

39. Consider the following statements :

In ammonia refrigeration systems, oil separator is provided because

1. oil separation in evaporator would lead to reduction in heat transfer coefficient.
2. oil accumulation in the evaporator causes choking of evaporator.
3. oil is partially miscible in the refrigerant.
4. oil causes choking of expansion device.

Of these statements :

(*a*) 1 and 2 are correct (*b*) 2 and 4 are correct

(*c*) 2, 3 and 4 are correct (*d*) 1, 3 and 4 are correct.

40. Consider the following statements :

Moisture should be removed from refrigerants to avoid :

1. compressor seal failure
2. freezing at the expansion valve
3. restriction to refrigerant flow
4. corrosion of steel parts.

On these statements :

(*a*) 1, 2, 3 and 4 are correct (*b*) 1 and 2 are correct

(*c*) 2, 3 and 4 are correct (*d*) 1, 3 and 4 are correct.

41. Consider the following statements :

1. Practically all common refrigerants have approximately the same COP and power requirement.
2. Ammonia mixes freely with lubricating oil and this helps lubrication of compressors.

3. Dielectric strength of refrigerants is an important property in hermetically sealed compressor units.
4. Leakage of ammonia can be detected by halide torch method.

Of these statements :

(*a*) 1, 2 and 4 are correct (*b*) 2 and 4 are correct
(*c*) 1, 2 and 4 are correct (*d*) 1 and 3 are correct.

42. The most commonly used method for the design of duct size is the
(*a*) velocity reduction method (*b*) equal friction method
(*c*) static regain method (*d*) dual or double method.

43. The refrigerant used for absorption refrigerators working on heat from solar collectors is a mixture of water and
(*a*) carbon dioxide (*b*) sulphur dioxide
(*c*) lithium bromide (*d*) freon 12.

44. During the adiabatic cooling of moist air
(*a*) DBT remains constant (*b*) specific humidity remains constant
(*c*) relative humidity remains constant (*d*) WBT remains constant.

45. When a stream of moist air is passed over a cold and dry cooling coil such that no condensation takes place, then the air stream will get cooled along the line of
(*a*) constant wet bulb temperature (*b*) constant dew point temperature
(*c*) constant relative humidity (*d*) constant enthalpy.

46. For large tonnage (more than 200 tons) air-conditioning applications, which one of the following types of compressors is recommended ?
(*a*) Reciprocating (*b*) Rotating
(*c*) Centrifugal (*d*) Screw.

47. In a cooling tower, "approach" is the temperature difference between the
(*a*) hot inlet water and cold outlet water (*b*) hot inlet water and WBT
(*c*) cold outlet water and WBT (*d*) DBT and WBT.

48. When the discharge pressure is too high in a refrigeration system, high pressure control is installed to
(*a*) stop the cooling fan (*b*) stop the water circulating pump
(*c*) regulate the flow of cooling water (*d*) stop the compressor.

49. A refrigerating machine working on reversed Carnot cycle takes out 2 kW per minute of heat from the system while working between temperature limits of 300 K and 200 K. C.O.P. and Power consumed by the cycle will be respectively
(*a*) 1 and 1 kW (*b*) 1 and 2 kW
(*c*) 2 and 1 kW (*c*) 2 and 2 kW.

50. Consider the following statements :

In the case of a vapour compression machine, if the condensing temperature of the refrigerant is closer to the critical temperature, then there will be

1. excessive power consumption 2. high compression
3. large volume flow.

Of these statements :

(*a*) 1, 2 and 3 are correct (*b*) 1 and 2 are correct
(*c*) 2 and 3 are correct (*d*) 1 and 3 are correct.

51. Hydrogen is essential in an Electrolux refrigeration system, because
(*a*) it acts as a catalyst in the evaporator
(*b*) the reaction between hydrogen and ammonia is endothermic in evaporator and exothermic in absorber
(*c*) the cooled hydrogen leaving the heat exchanger cools the refrigerant entering the evaporator
(*d*) it helps in maintaining a low partial pressure for the evaporating ammonia.

52. In an ideal refrigeration (reversed Carnot) cycle, the condenser and evaporator temperatures are 27°C and – 13°C respectively. The COP of this cycle would be
(*a*) 6.5 (*b*) 7.5
(*c*) 10.5 (*d*) 15.0.

53. A single-stage vapour compression refrigeration system cannot be used to produce ultra low temperatures because
(*a*) refrigerants for ultra-low temperatures are not available
(*b*) lubricants for ultra-low temperatures are not available
(*c*) volumetric efficiency will decrease considerably
(*d*) heat leakage into the system will be excessive.

54. Vapour absorption refrigeration system works using the
(*a*) ability of a substance to get easily condensed or evaporated
(*b*) ability of a vapour to get compressed or expanded
(*c*) affinity of a substance for another substance
(*d*) absorptivity of a substance.

55. Which one of the following statements regarding ammonia absorption system is correct ?
The solubility of ammonia in water is
(*a*) a function of the temperature and pressure of the solution
(*b*) a function of the pressure of the solution irrespective of the temperature
(*c*) a function of the temperature of the solution alone
(*d*) independent of the temperature and pressure of the solution.

56. Consider the following statements :
In thermoelectric refrigeration, the coefficient of performance is a function of
1. electrical conductivity of materials.
2. Peltier coefficient.
3. Seebeck coefficient.
4. temperature at cold and hot junctions.
5. thermal conductivity of materials.
Of these statements :
(*a*) 1, 3, 4 and 5 are correct (*b*) 1, 2, 3 and 5 are correct
(*c*) 1, 2, 4 and 5 are correct (*d*) 2, 3, 4 and 5 are correct.

57. Air cooling is used for freon compressors whereas water jacketing is adopted for cooling ammonia compressors. This is because
(*a*) latent heat of ammonia is higher than that of a freon
(*b*) thermal conductivity of water is higher than that of air
(*c*) specific heat of water is higher than that of air
(*d*) of the larger superheat horn of ammonia compression cycle.

58. Consider the following statements :

A psychrometer measures

1. wet bulb temperature
2. dew point temperature
3. dry bulb temperature.

Of these statements :

(*a*) 1 alone is correct (*b*) 2 and 3 are correct

(*c*) 1 and 3 are correct (*d*) 1, 2 and 3 are correct.

59. Hot coffee in a cup is allowed to cool. Its cooling rate is measured and found to be greater than the value calculated by conduction, convection and radiation measurements. The difference is due to

(*a*) properties of coffee changing with temperature

(*b*) currents of air flow in the room (*c*) underestimation of the emissivity of coffee

(*d*) evaporation.

60. For an air-conditioning plant above 300 ton, which one of the following systems would normally be preferred ?

(*a*) Ammonia reciprocating compressor (*b*) Centrifugal chiller

(*c*) Absorption refrigeration system (*d*) Hermetic compressor.

61. Fresh air intake (air change per hour) recommended for ventilation purposes in the air-conditioning system of an office building is

(*a*) 1/2 (*b*) 3/2

(*c*) 9/2 (*d*) 25/2.

62. Give that

Nu = Nusselt number, Re = Reynolds number,

Pr = Prandtl number, Sh = Sherwood number,

Sc = Schmidt number, and Gr = Grashoff number,

the functional relationship for free convective mass transfer is given as :

(*a*) Nu = f (Gr, Pr) (*b*) Sh = f (Sc, Gr)

(*c*) Nu = f (Re, Pr) (*d*) Sh = f (Re, Sc).

63. Air refrigeration cycle is used in

(*a*) commercial refrigerators (*b*) domestic refrigerators

(*c*) gas liquification (*d*) air-conditioning.

64. The flash chamber in single-stage simple vapour compression cycle

(*a*) increases the refrigerating effect (*b*) decrease the refrigerating effect

(*c*) increases the work of compression (*d*) has no effect on refrigerating effect.

65. Consider the following statements :

In a vapour compression system, a thermometer placed in the liquid line can indicate whether the

1. refrigerant flow is too low
2. water circulation is adequate
3. condenser is fouled
4. pump is functioning properly.

Of these statements :

(*a*) 1, 2 and 3 are correct (*b*) 1, 2 and 4 are correct

(*c*) 1, 3 and 4 are correct (*d*) 2, 3 and 4 are correct.

66. Match List with List II and select the correct answer using the codes given below the Lists :

List I	**List II**
A. Bell Colemen refrigeration	1. Compressor
B. Vapour compression refrigeration	2. Generator
C. Absorption refrigeration	3. Flash chamber
D. Jet refrigeration	4. Expansion cylinder

Codes :

	A	B	C	D
(*a*)	1	4	3	2
(*b*)	4	1	3	2
(*c*)	1	4	2	3
(*d*)	4	1	2	3.

67. The maximum C.O.P. for the absorption cycle is given by (T_G = generator temperature, T_C = environment temperature, T_E = refrigerated space temperature)

(*a*) $\dfrac{T_E(T_G - T_C)}{T_G(T_C - T_E)}$ (*b*) $\dfrac{T_G(T_C - T_E)}{T_E(T_G - T_C)}$

(*c*) $\dfrac{T_C(T_G - T_E)}{T_G(T_C - T_E)}$ (*d*) $\dfrac{T_G(T_C - T_E)}{T_C(T_G - T_E)}$.

68. In milk chilling plants, the usual secondary refrigerant is

(*a*) ammonia solution (*b*) sodium silicate

(*c*) glycol (*d*) brine.

69. The desirable combination of properties for a refrigerant include

(*a*) high specific heat and low specific volume

(*b*) high heat transfer coefficient and low latent heat

(*c*) high thermal conductivity and low freezing point

(*d*) high specific heat and high boiling point.

70. Which of the following method(s) is/are adopted in the design of air duct system ?

1. Velocity reduction method 2. Equal friction method 3. Static regain method.

Select the correct answer using the codes given below :

Codes :

(*a*) 1 alone (*b*) 1 and 2

(*c*) 2 and 3 (*d*) 1, 2 and 3.

71. To fix the state point in respect of air-vapour mixtures, three intrinsic properties are needed. Yet, the psychrometric chart requires only two because

(*a*) water vapour is in the superheated state

(*b*) the chart is for a given pressure

(*c*) the chart is an approximation to true values

(*d*) the mixtures can be treated as a perfect gas.

72. During sensible cooling of air,

(*a*) its wet bulb temperature increases and dew point remains constant

(*b*) its wet bulb temperature decreases and the dew point remains constant

(*c*) its wet bulb temperature increases and the dew point decreases

(*d*) its wet bulb temperature decreases and dew point increases.

73. The expression $\frac{0.622\, p_v}{p_1 - p_v}$ is used to determine

(*a*) relative humidity (*b*) specific humidity
(*c*) degree of saturation (*d*) partial pressure.

74. The effective temperature is a measure of the combined effects of

(*a*) dry bulb temperature and relative humidity
(*b*) dry bulb temperature and air motion
(*c*) wet bulb temperature and air motion
(*d*) dry bulb temperature, relative humidity and air motion

75. In air-conditioning design for summer months, the condition inside a factory where heavy work is performed as compared to a factory in which light work is performed should have

(*a*) lower dry bulb temperature and lower relative humidity
(*b*) lower dry bulb temperature and higher relative humidity
(*c*) lower dry bulb temperature and same relative humidity
(*d*) same dry bulb temperature and same relative humidity.

ANSWERS

1. (*c*)	**2.** (*b*)	**3.** (*c*)	**4.** (*d*)	**5.** (*a*)	**6.** (*d*)	**7.** (*c*)
8. (*a*)	**9.** (*b*)	**10.** (*a*)	**11.** (*a*)	**12.** (*c*)	**13.** (*a*)	**14.** (*c*)
15. (*b*)	**16.** (*d*)	**17.** (*b*)	**18.** (*a*)	**19.** (*a*)	**20.** (*c*)	**21.** (*a*)
22. (*c*)	**23.** (*d*)	**24.** (*c*)	**25.** (*b*)	**26.** (*a*)	**27.** (*b*)	**28.** (*c*)
29. (*c*)	**30.** (*b*)	**31.** (*c*)	**32.** (*a*)	**33.** (*b*)	**34.** (*d*)	**35.** (*d*)
36. (*c*)	**37.** (*d*)	**38.** (*b*)	**39.** (*d*)	**40.** (*c*)	**41.** (*d*)	**42.** (*c*)
43. (*c*)	**44.** (*d*)	**45.** (*b*)	**46.** (*c*)	**47.** (*c*)	**48.** (*d*)	**49.** (*c*)
50. (*b*)	**51.** (*d*)	**52.** (*a*)	**53.** (*d*)	**54.** (*c*)	**55.** (*a*)	**56.** (*b*)
57. (*c*)	**58.** (*a*)	**59.** (*b*)	**60.** (*c*)	**61.** (*b*)	**62.** (*c*)	**63.** (*d*)
64. (*a*)	**65.** (*a*)	**66.** (*d*)	**67.** (*a*)	**68.** (*d*)	**69.** (*d*)	**70.** (*d*)
71. (*b*)	**72.** (*a*)	**73.** (*b*)	**74.** (*d*)	**75.** (*a*).		

SOLUTIONS—COMMENTS

6. Because throttling is not actually an isothermal process. The expansion of gas causes a fall in temperature and increased kinetic energy increases it to the initial level.

13. For isothermal process, $T_1 = T_2$

or $$p_1V_1 = p_2V_2 \;\Rightarrow\; V_1 = \frac{p_2V_2}{p_1}$$

Given : $$\frac{p_1}{p_2} = 10 \text{ and } V_2 = 0.55 \text{ m}^3$$

$\therefore$ $$V_1 = \frac{0.55}{10} = 0.055 \text{ m}^3$$

For adiabatic expansion, pV^γ = constant

or $$p_1V_1^\gamma = p_2V_2^\gamma \;\Rightarrow\; V_2 = \left(\frac{p_1}{p_2}\right)^{\frac{1}{\gamma}} \times V_1 = (10)^{\frac{1}{1.4}} \times 0.055 = 0.2848 \text{ m}^3$$

14. $$T_1 = T_2 \; ; \; V_2 = \frac{V_1}{2} \quad \text{...(Given)}$$

From $$p_1V_1 = p_2V_2, \quad p_1V_1 = p_2 \times \frac{V_1}{2} \quad \Rightarrow \quad \mathbf{p_2 = 2p_1}$$

15. Since $(\Delta s)_{system} > (\Delta s)_{surrounding}$, where both $(\Delta s)_{system} > 0$ and $(\Delta s)_{surrounding} > 0$.

40. The compressors seal cannot fail due to moisture, all the other conditions do occur due to presence of moisture.

44. All common refrigerants like F_{11}, F_{12}, F_{22}, NH_3 etc. have approximately the same C.O.P. ranging from 4.76 to 5.09 and H.P./ton varies from 0.99 to 1.01.

The electric resistance of the refrigerants is an important factor when it is used in hermetically sealed unit where the motor is exposed to the refrigerant.

46. The reason why centrifugal compressors are used to large tonnage is that they can handle larger volumes of refrigerant, also the part load efficiency of this kind is higher.

49. $$\text{C.O.P.} = \frac{T_2}{T_1 - T_2} = \frac{200}{300 - 200} = 2$$

$$\text{Power consumed} = \frac{R_n \text{ (net refrigerating effect)}}{\text{C.O.P.}} = \frac{2 \text{ kW}}{2} = 1 \text{ kW}.$$

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